

EECS 498 System Design of a Search Engine

Winter 2026

Lecture 23: Time value of money

Nicole Hamilton

[https://web.eecs.umich.edu/~nham/
nham@umich.edu](https://web.eecs.umich.edu/~nham/nham@umich.edu)

agenda

1. details
2. time value of money

agenda

1. details
2. time value of money

details

1. I have posted guidance on your presentations to Canvas in the Files > Assignments folder.
2. I have several mixed teams. Please verify I know what section your team is in for scheduling your presentation.
3. I am working on guidance for your final reports.
4. Course evals have opened and stay open until Wed Apr 22.
5. Peer eval forms posted to Canvas, due Tue Apr 21.

remaining schedule

1. Today: Time value of money
2. Thu April 9: Reflections on the course
3. Tue April 14: Review for the final
4. Thu April 17, Fri April 18, Tue April 21: Presentations
5. Tues Apr 21: Peer evals, final reports and project archive due
6. Wed April 22 through Wed April 29: Team debriefs
7. Wed April 29 4:00pm: Optional final exam

agenda

1. details
2. time value of money

time value of money

I first encountered this my last year at Stanford as “Engineering Economy”.

As a skill, I’ve found it to be right up there with typing.

we'll answer questions like this

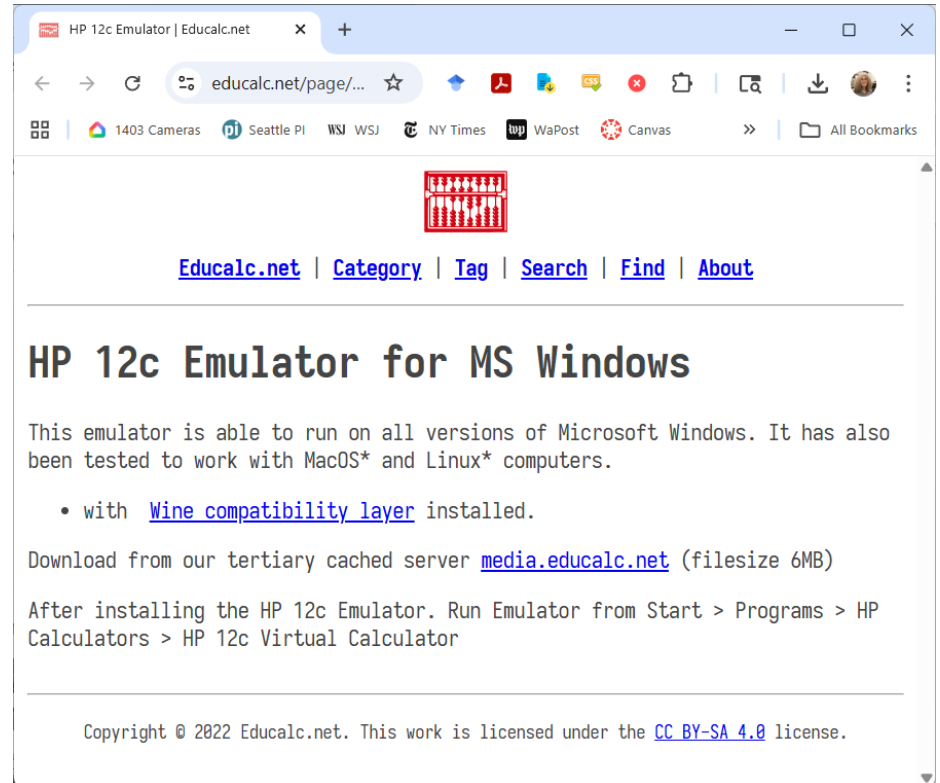
Ed McMahon has risen from the dead and is standing on your doorstep, offering you a check for \$1,000,000 now or \$10,000/month for 20 years. Tax-free.

Which is the better deal?

you'll need an HP-12 emulator



Fortunately, HP released a free one.
<https://www.educalc.net/page/2338917/>



I also have a few physical HP-12s.

A dollar today is worth more than a dollar promised tomorrow.

We discount the future.

terminology

Interest is money paid for the use of borrowed money.

Principal is the amount borrowed.

Interest rate is the interest payable at the *end* of a period divided by the money owed at the *beginning* of a period. Usually expressed as a %.

compounding

Simple interest is calculated by multiplying the interest rate $\times$ principal $\times$ number of periods.

Compound interest is interest added to the principal at the end of each period so that it also earns interest.

example

You borrow \$1000 at 6%/year for two years.
How much do you owe at the end?

Simple interest:

$$\$1000 * (1 + .06 * 2) = \$1120$$

Compound interest:

$$\text{1st year: } \$1000 * (1 + .06) = \$1060$$

$$\text{2nd year: } \$1060 * (1 + .06) = \$1123 \leftarrow \text{An extra \$3}$$

\$1 invested at 7% doubles every 10 years.

the power of compounding

Manhattan was supposedly purchased by European explorers for \$24 in 1626.

If that \$24 had been continuously invested at 7%, it would now be worth $\$6.04 \times 10^{12} = \$6.04$ trillion.

The actual estimated current value of Manhattan is about \$3 trillion.

reality check

All loans you will ever encounter will always use compound interest.

They will state it as an APR.

APR

Annual Percentage Rate. A term introduced by the *Truth in Lending Act* of 1968 which standardized disclosure of lending costs.

This is the interest rate for a loan expressed as an annual rate.
This is the ***nominal*** rate.

*But the compounding period is usually **monthly** meaning the **effective** interest rate is higher.*

converting nominal to effective

Effective APR =

$$(1 + \text{Rate per period})^{\text{Periods per year}} - 1$$

Effective APR =

$$(1 + \text{Rate per period})^{\text{Periods per year}} - 1$$

Examples

$$\text{Nominal APR} = 12\% = 0.12$$

$$\text{Rate per period} = 1\% = 0.01$$

$$\text{Effective APR} = (1 + 0.01)^{12} - 1 = 0.1268 = 12.68\%$$

$$\text{Nominal APR} = 18\% = 0.18$$

$$\text{Rate per period} = 1.5\% = 0.015$$

$$\text{Effective APR} = (1 + 0.015)^{12} - 1 = 0.1956 = 19.56\%$$

concept of equivalence

Payments that differ in magnitude but are made *at different dates* may be financially ***equivalent*** when discounted at an interest rate.

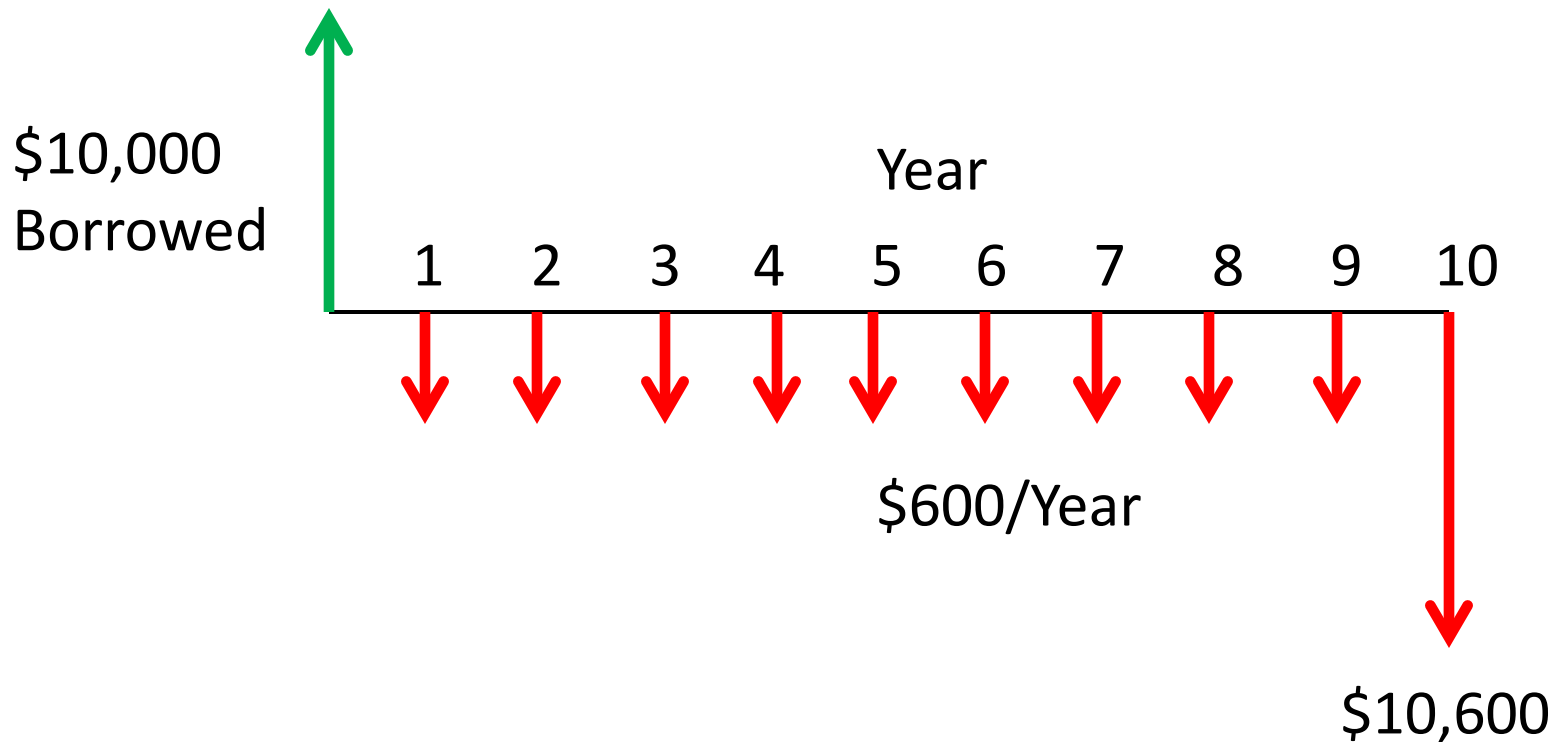
example

You borrow \$10,000 for 10 years at 6% compounded annually.

Any number of repayment schedules might be *equivalent*.

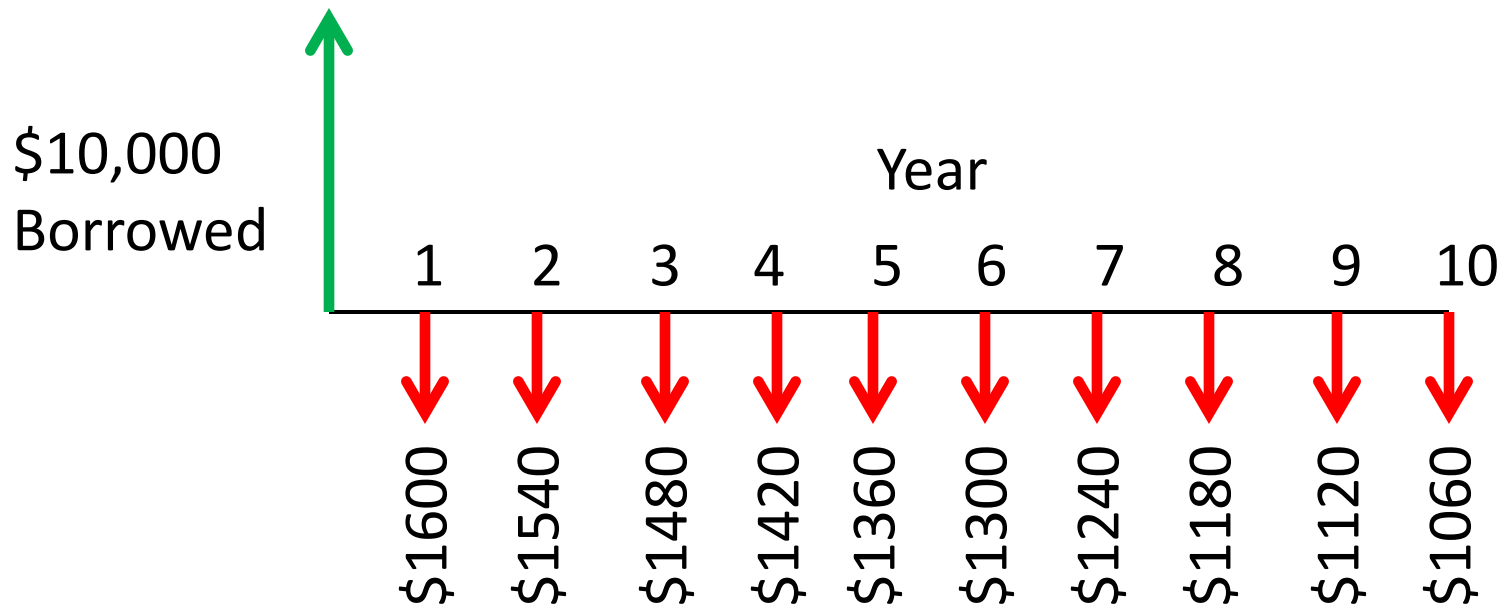
To examine them, we use ***cash flow diagrams***.

interest only + balloon at the end

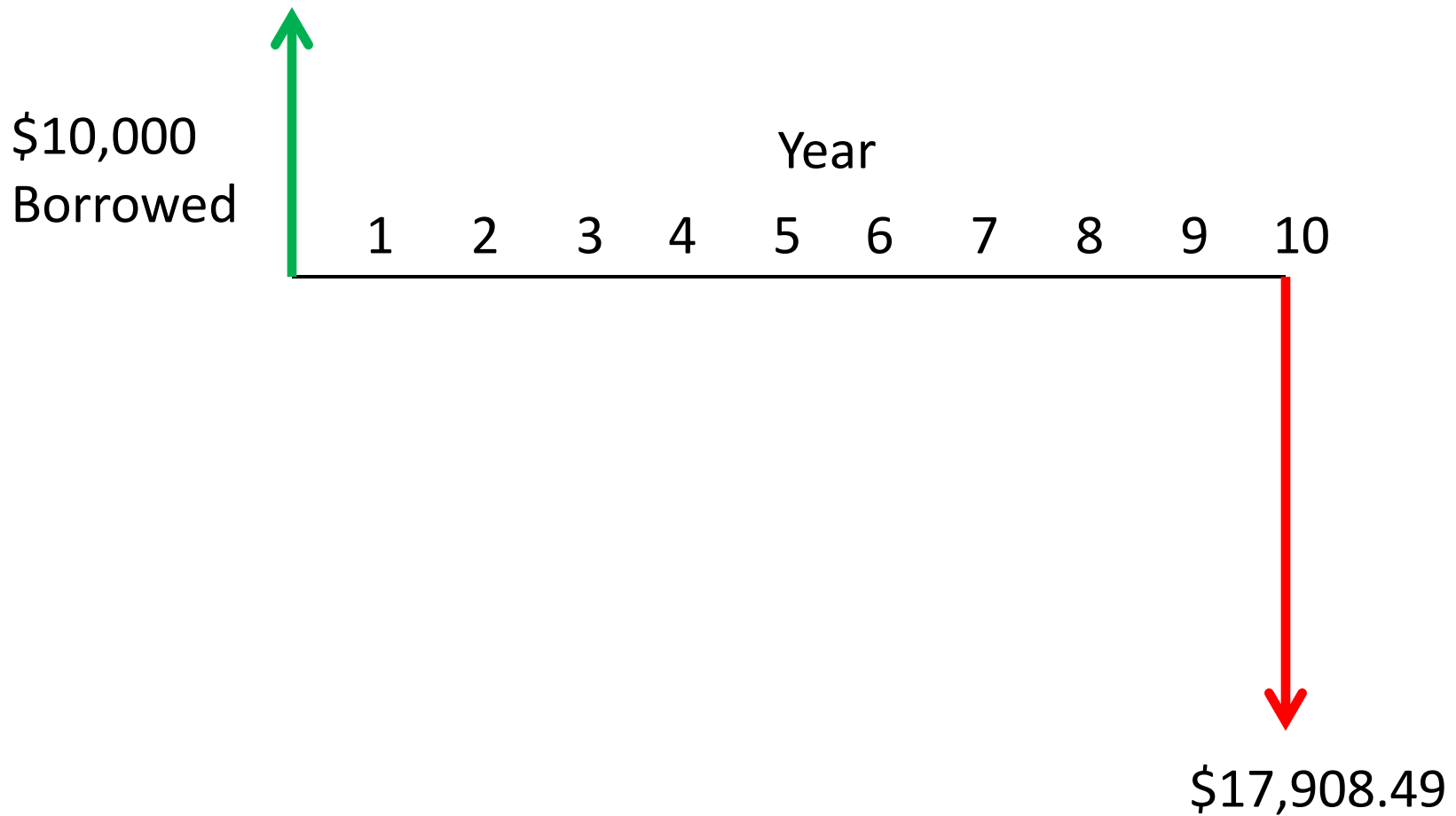


Source: Grant & Ireson, *Principles of Engineering Economy*, 5th Ed., pg. 28.

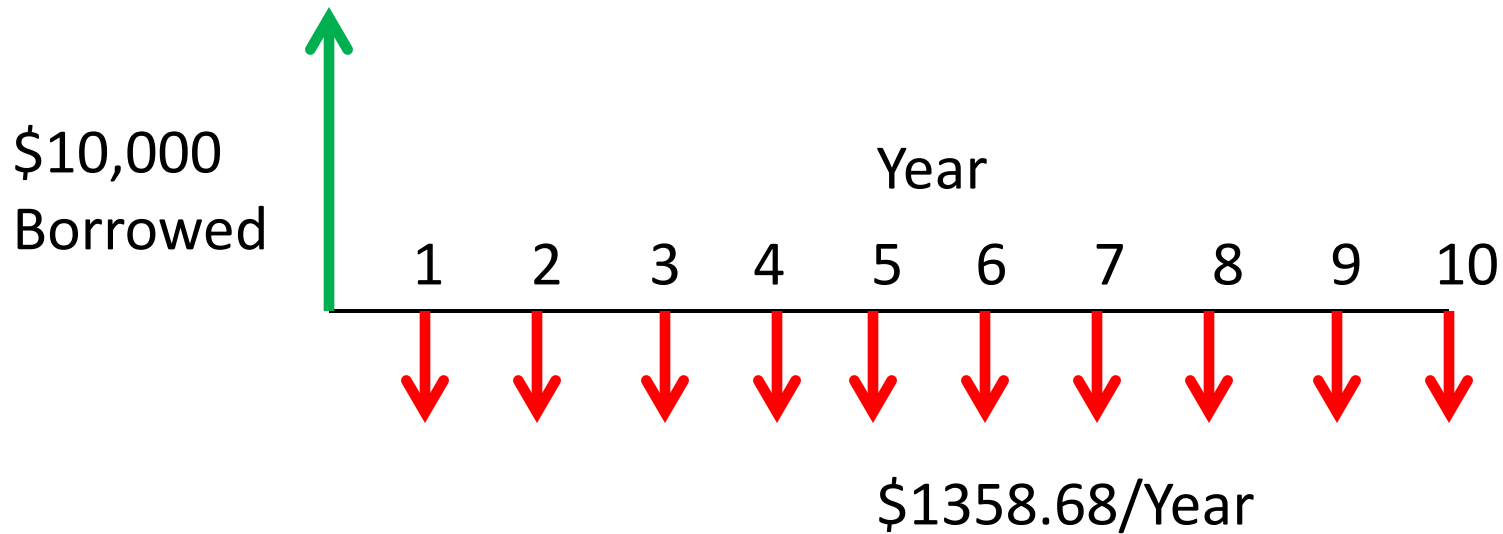
fixed principal + interest



single payment



an annuity



A series of equal payments at the end of each period is called an ***annuity***.

terms

Present value is the value today of any series of discounted payments.

Future value is the value at some future date of any series of discounted payments.

P = Present value

F = Future value

A = Annuity amount at each period

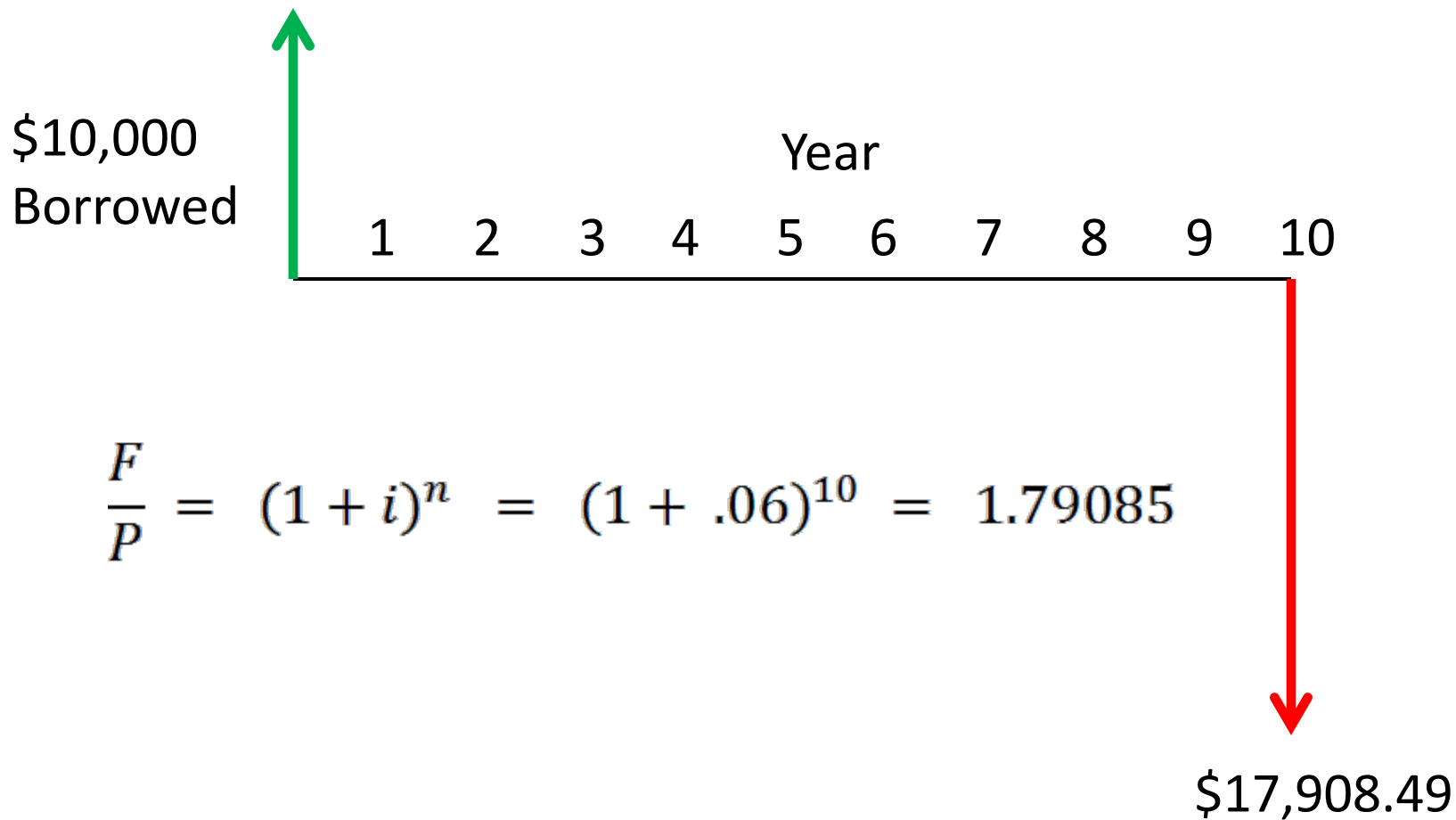
n = Number of periods

i = Interest rate per period

present and future

$$\frac{F}{P} = (1 + i)^n$$

$$\frac{P}{F} = \frac{1}{(1 + i)^n}$$



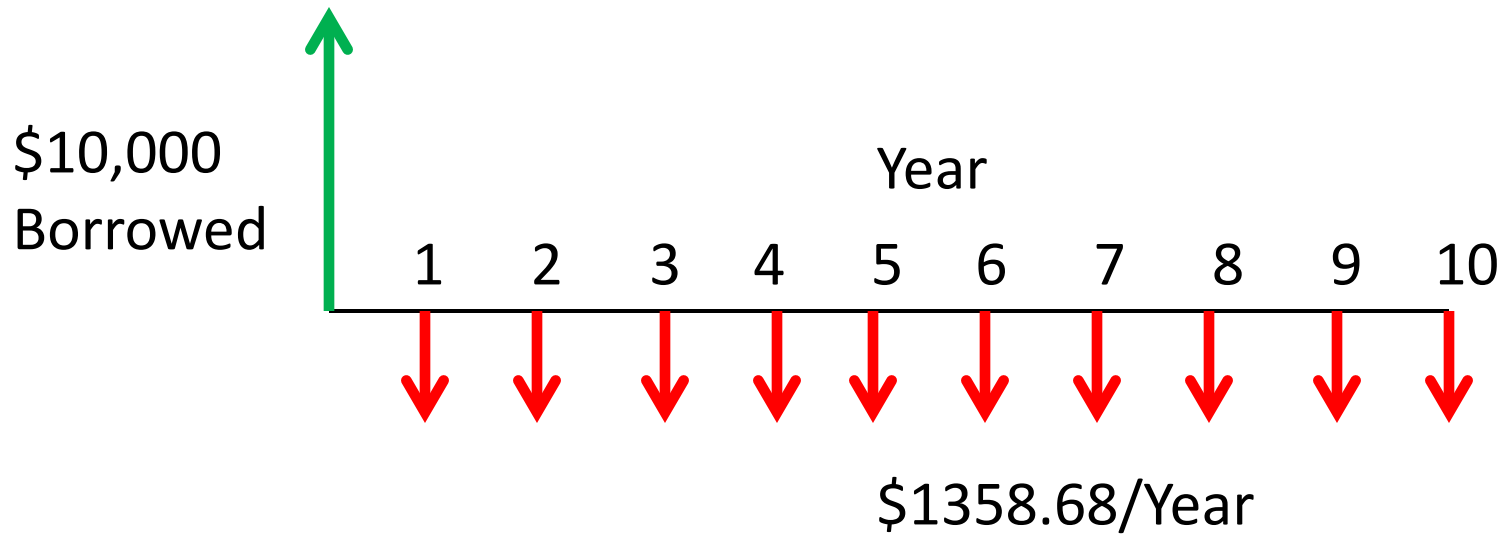
annuities

$$\frac{P}{A} = \frac{(1+i)^n - 1}{i(1+i)^n}$$

$$\frac{A}{P} = \frac{i(1+i)^n}{(1+i)^n - 1}$$

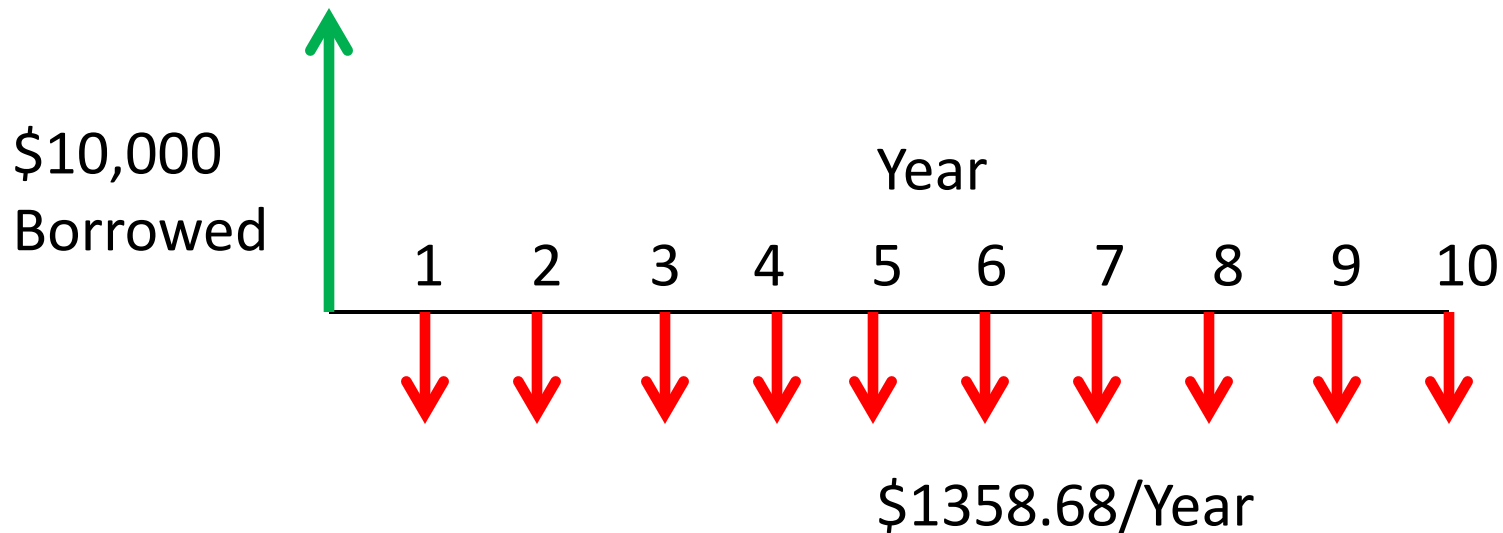
$$\frac{F}{A} = \frac{(1+i)^n - 1}{i}$$

$$\frac{A}{F} = \frac{i}{(1+i)^n - 1}$$



$$\frac{A}{P} = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{(.06)(1.06)^{10}}{(1.06)^{10} - 1} = 0.13587$$

derivation of P/A



The present value of an annuity is the sum of the present values of the individual payments.

$$\frac{P}{A} = \sum_{j=1}^n \frac{1}{(1+i)^j}$$

This is a *power series*.

power series

$$S = \sum_{j=1}^n \frac{1}{r^j} = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \dots + \frac{1}{r^n}$$

$$rS = 1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \dots + \frac{1}{r^{n-1}}$$

$$rS - S = S(r - 1) = 1 - \frac{1}{r^n}$$

$$S = \frac{1}{r - 1} - \frac{1}{r^n(r - 1)} = \frac{r^n - 1}{r^n(r - 1)}$$

The present value of an annuity is the sum of the present values of the individual payments.

$$\frac{P}{A} = \sum_{j=1}^n \frac{1}{(1+i)^j}$$

Use the *power series* rule.

$$s = \sum_{j=1}^n \frac{1}{r^j} = \frac{r^n - 1}{r^n(r - 1)}$$

$$s = \frac{P}{A} \quad r = 1 + i$$

Therefore,

$$\frac{P}{A} = \frac{(1+i)^n - 1}{(1+i)^n(1+i-1)} = \frac{(1+i)^n - 1}{i(1+i)^n}$$

calculating i and n

Given a conversion factor,

Easy to calculate n if you know i .

Easy to calculate i if you know n and F/P or P/F but not so easy for annuities.

calculating i

Easy enough from F/P :

$$\frac{F}{P} = (1 + i)^n$$

$$\left(\frac{F}{P}\right)^{\frac{1}{n}} = 1 + i$$

$$i = \left(\frac{F}{P}\right)^{\frac{1}{n}} - 1$$

And because P/F is just the inverse of F/P :

$$i = \left(\frac{P}{F}\right)^{-\frac{1}{n}} - 1$$

calculating n

Easy enough from F/P:

$$\frac{F}{P} = (1 + i)^n$$

$$\log\left(\frac{F}{P}\right) = n \log(1 + i)$$

$$n = \frac{\log\left(\frac{F}{P}\right)}{\log(1 + i)}$$

And because P/F is just the inverse of F/P:

$$n = \frac{-\log\left(\frac{F}{P}\right)}{\log(1 + i)}$$

calculating n

And from F/A :

$$\frac{F}{A} = \frac{(1+i)^n - 1}{i}$$

$$i\left(\frac{F}{A}\right) = (1+i)^n - 1$$

$$i\left(\frac{F}{A}\right) + 1 = (1+i)^n$$

$$\log\left(i\left(\frac{F}{A}\right) + 1\right) = n \log(1+i)$$

$$n = \frac{\log\left(i\left(\frac{F}{A}\right) + 1\right)}{\log(1+i)}$$

No simple algebraic way to solve for i for an annuity, given F/A and n .

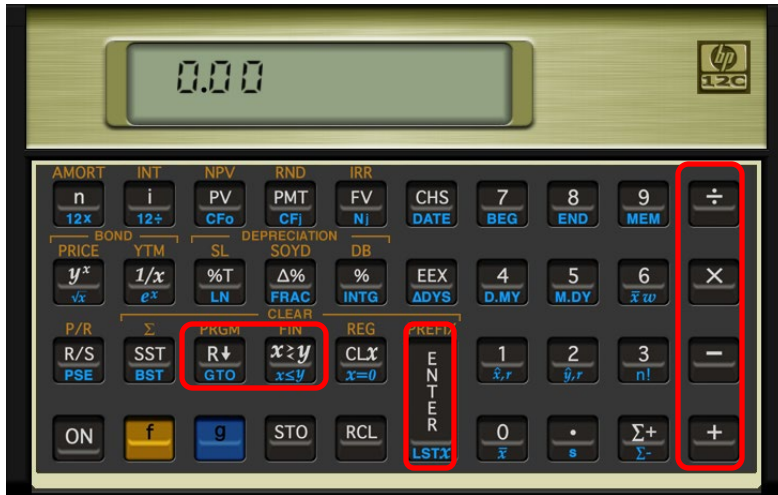
$$\frac{F}{A} = \frac{(1+i)^n - 1}{i}$$

Usual approach is to solve numerically using an iterative approach of making closer and closer estimates.

the workhorse HP-12c



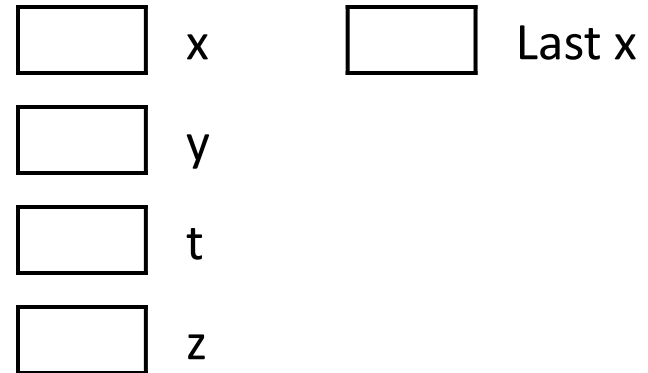
reverse Polish



You enter the operands into the stack,
then operate on the top elements.

Instead of $5 + 2 =$
you type 5 ENTER 2 +

Stack registers



x is always displayed.
Whatever you type is in the x register.
Last value of x always kept in Last x.

Operations:

	Push	ENTER
Rotate	R↓	
Exchange	x↔y	
Arithmetic	÷ − × +	

shift keys



f + 0 thru 9 sets number of decimal places.

f + . sets scientific notation.

(Unfortunately, they don't light up on the real thing.)

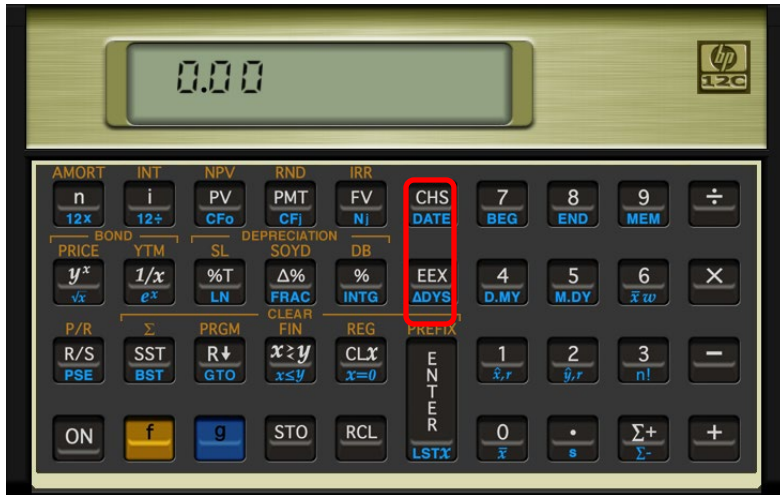
clearing registers



f + FIN clears the financial registers.

f + REG clears the stack, Last x and the financial registers.

entering numbers



EEX allows you to type scientific notation.

CHS changes the sign of an exponent or of a number *already* typed.

To get this

Type this

-5

5 CHS

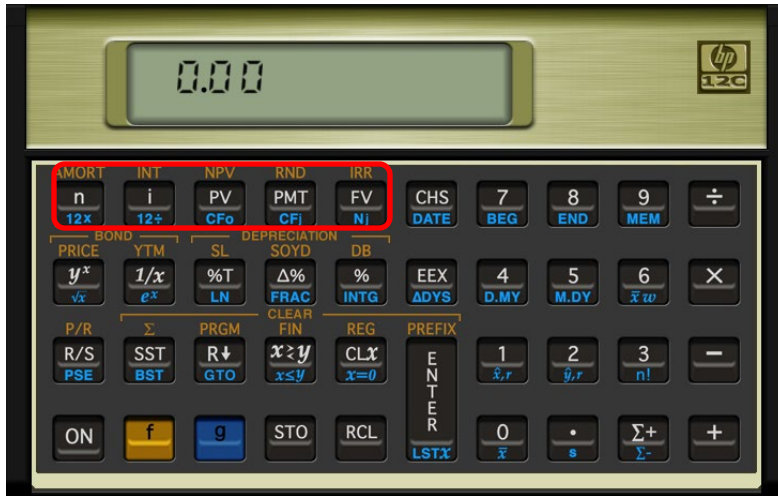
2E6

2 EEX 6

-7E-3

7 CHS EEX 3 CHS

financial registers



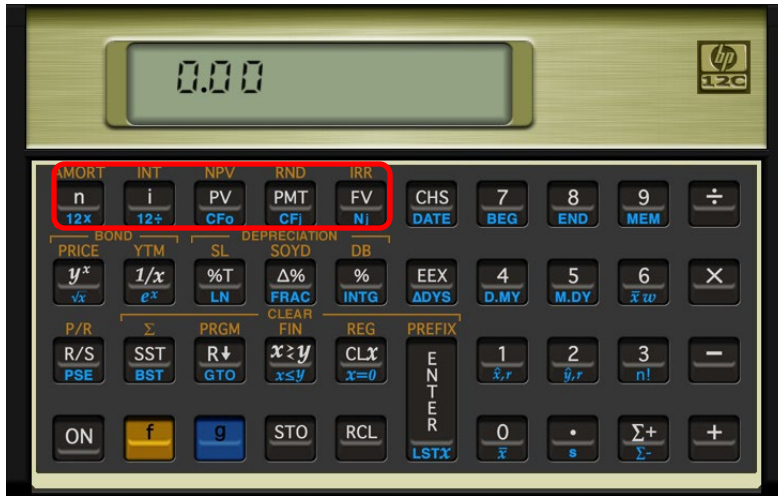
n = number of periods
 i = interest rate as a %
 PV = present value
 PMT = annuity amount
 FV = future value

Financial registers

	n		$N0$		$CF0 (R0)$
	i		$N1$		$CF1 (R1)$
	PV		$N2$		$CF2 (R2)$
	PMT		$N3$		$CF3 (R3)$
	FV		$N4$		$CF4 (R4)$
		:		:	
			$N20$		$CF20 (R.9)$

$N1$ thru $N20$ and $CF0$
 thru $CF20$ are for
 irregular cash flows,
 where CF_i is repeated N_i
 times.

financial registers



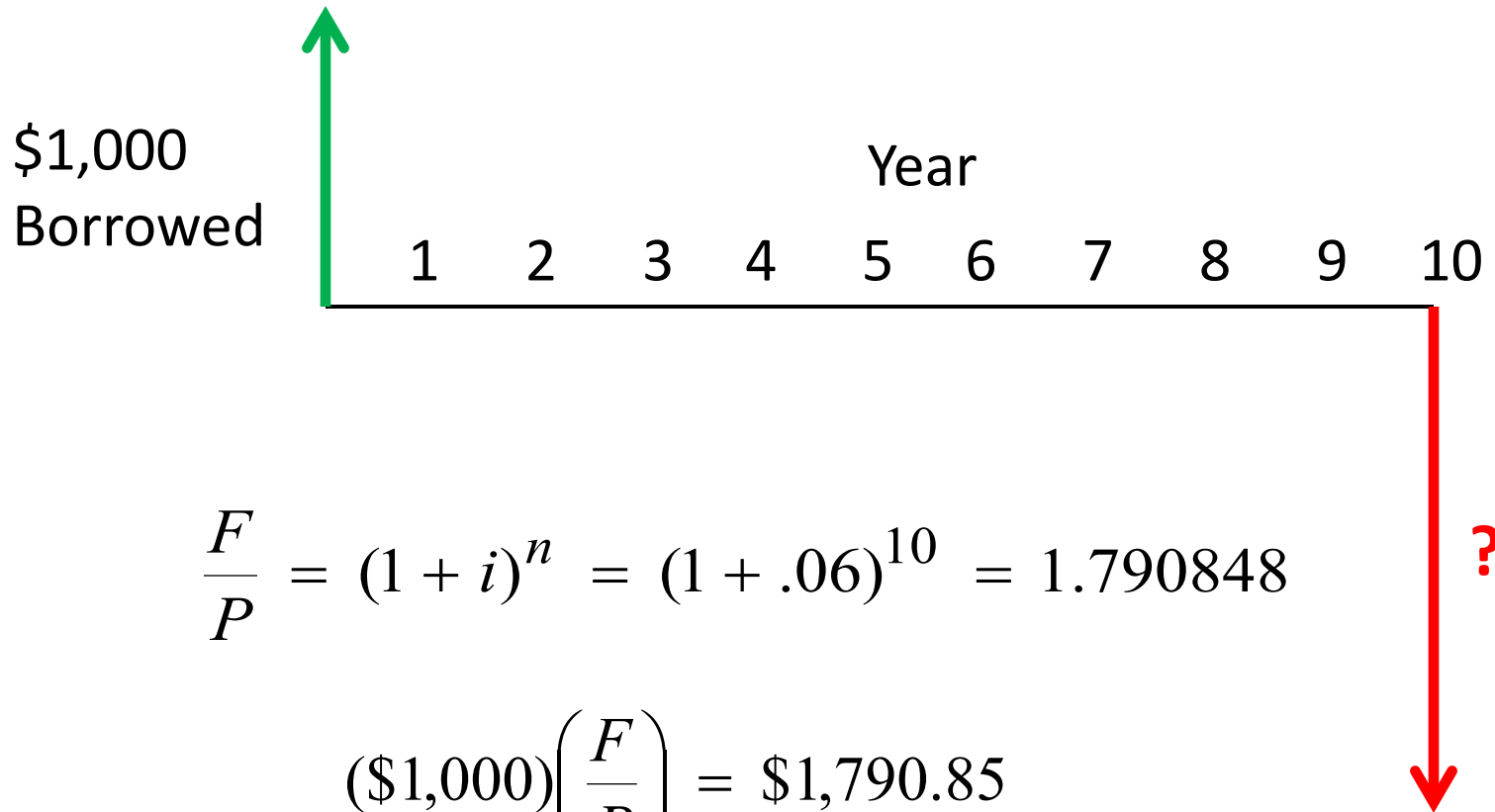
Financial registers

	n		N0		CF0 (R0)
	i		N1		CF1 (R1)
	PV		N2		CF2 (R2)
	PMT		N3		CF3 (R3)
	FV		N4		CF4 (R4)
	:		:		
			N20		CF20 (R.9)

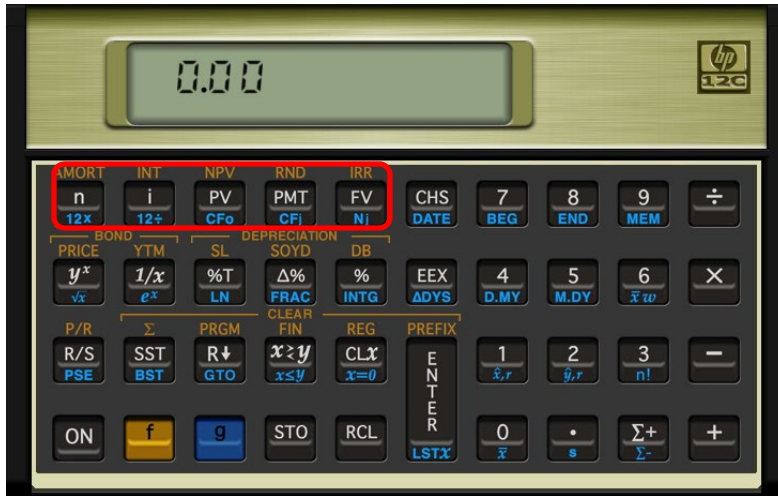
Typing a number and pressing n, i, PV, PMT or FV *stores* that value.

Pressing n, i, PV, PMT or FV alone *calculates* the value based on what's in the other registers.

example: \$1000 borrowed for 10 years
at 6% per year, compounded annually.

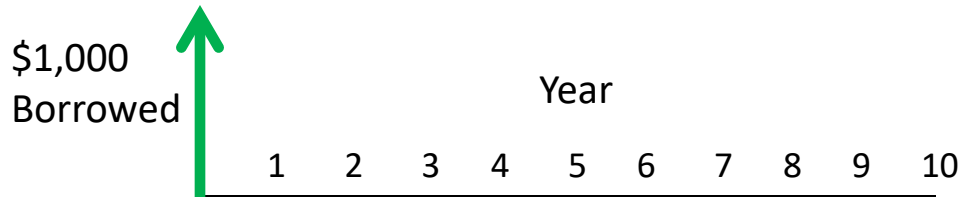


example: \$1000 borrowed for 10 years at 6% per year.



Financial registers

10	n		N0		CF0 (R0)
6	i		N1		CF1 (R1)
1000	PV		N2		CF2 (R2)
0	PMT		N3		CF3 (R3)
?	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



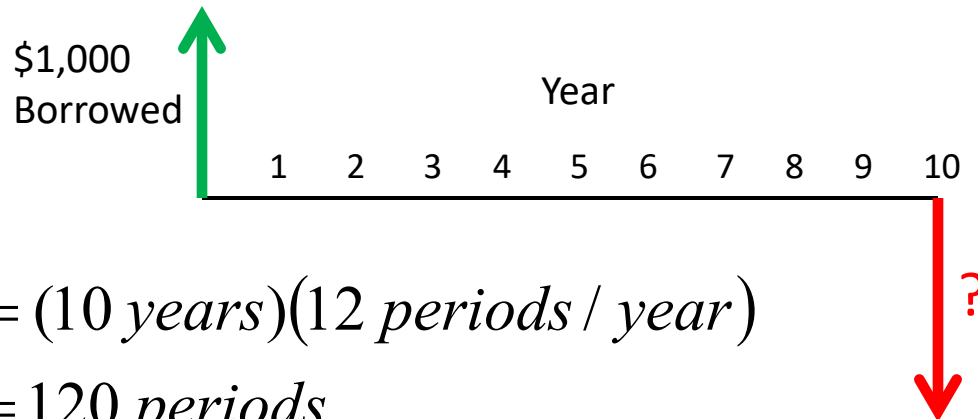
The result from the formula was \$1,790.85.
What does the HP-12c say?

Procedure:

1. Clear the registers.
2. Enter n, i and PV.
3. Press FV.

But interest rates on loans in the US are stated as APRs.

example: \$1000 borrowed for 10 years at 6% **APR**



$$n = (10 \text{ years})(12 \text{ periods / year})$$
$$= 120 \text{ periods}$$

$$i = \frac{.06}{12} = .005$$

$$\frac{F}{P} = (1 + i)^n = (1 + .005)^{120} = 1.819397$$

$$(\$1,000) \left(\frac{F}{P} \right) = \$1,819.40$$

example: \$1000 borrowed for 10 years at 6% APR.



Financial registers

120	n		N0		CF0 (R0)
.5	i		N1		CF1 (R1)
1000	PV		N2		CF2 (R2)
0	PMT		N3		CF3 (R3)
?	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



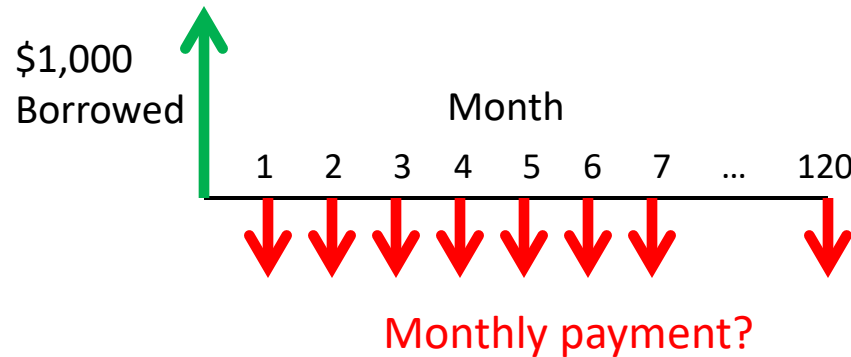
The result from the formula was \$1,819.40.
What does the HP-12c say?

Procedure:

1. Clear the registers.
2. Enter n and i using the 12x and 12÷ functions.
3. Enter PV.
4. Press FV.

And most loans in the US require monthly payments.

example: \$1000 borrowed for 10 years
at 6% APR with monthly payments



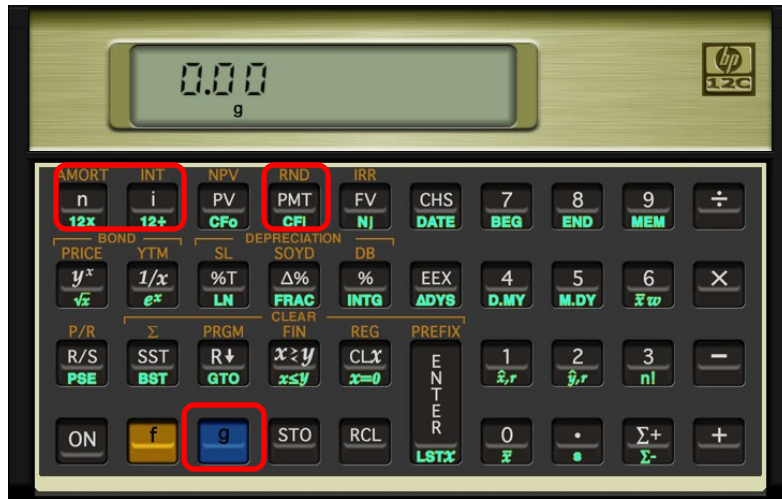
$$n = 120$$

$$i = .005$$

$$\frac{A}{P} = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{(.005)(1+.005)^{120}}{(1+.005)^{120} - 1} = .011102$$

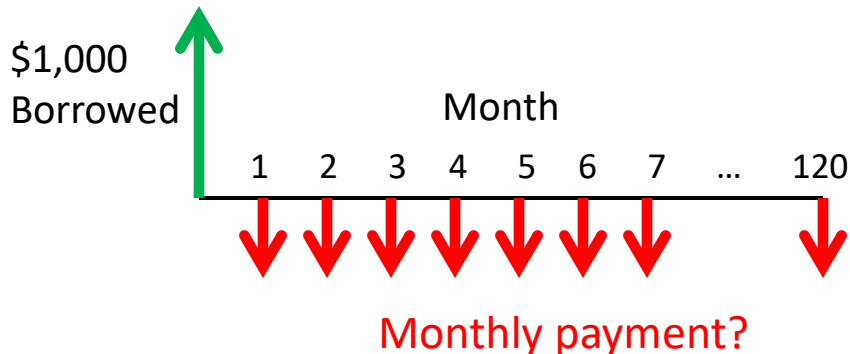
$$(\$1000) \left(\frac{A}{P} \right) = \$11.10$$

example: \$1000 borrowed for 10 years
at 6% APR with monthly payments.



Financial registers

120	n		N0		CF0 (R0)
.5	i		N1		CF1 (R1)
1000	PV		N2		CF2 (R2)
?	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



The result from the formula was \$11.10.
What does the HP-12c say?

Procedure:

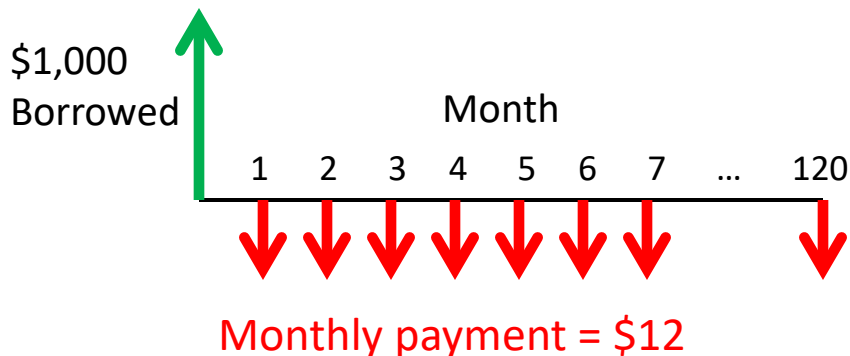
1. Clear the registers.
2. Enter n and i using the 12x and 12÷ functions.
3. Enter PV.
4. Press PMT.

example: \$1000 borrowed for 10 years
with monthly payments of \$12. What's the APR?



Financial registers

120	n		N0		CF0 (R0)
?	i		N1		CF1 (R1)
1000	PV		N2		CF2 (R2)
-12	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Procedure:

1. Clear the registers.
2. Enter n using the 12x function.
3. Enter PV and PMT
4. Press i.
5. Multiply times 12.

exercise

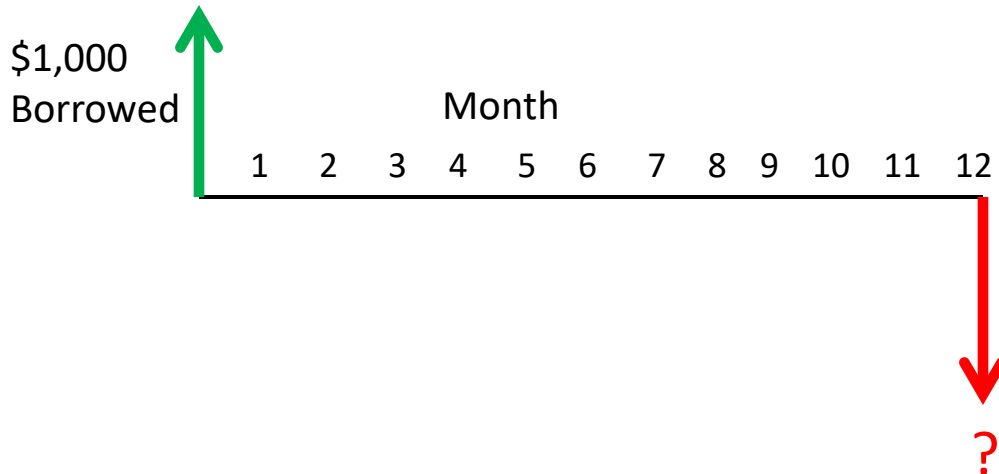
You owe \$1000 at 29.99% APR. If you make no payments, what will you owe a year from now?

\$1000 borrowed for 1 year at 29.99% APR



Financial registers

12	n		N0		CF0 (R0)
2.4992	i		N1		CF1 (R1)
1000	PV		N2		CF2 (R2)
0	PMT		N3		CF3 (R3)
?	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Procedure:

1. Clear the registers.
2. Enter n and i using the 12x and 12÷ functions.
3. Enter PV.
4. Press FV.

exercise

Ten years ago, Acme Corporation financed a new warehouse in part with a mortgage for \$500K for 20 years at 5%.

What's their monthly mortgage payment?

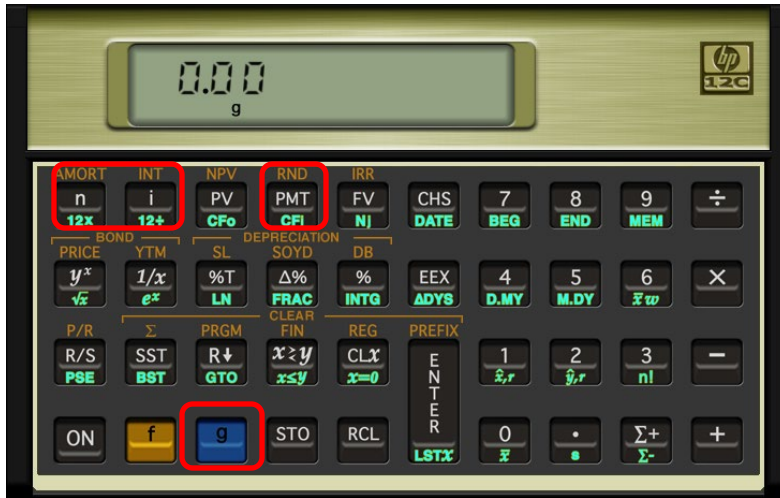
exercise

Ten years ago, Acme Corporation financed a new warehouse in part with a mortgage for \$500K for 20 years at 5%.

What's their monthly mortgage payment?

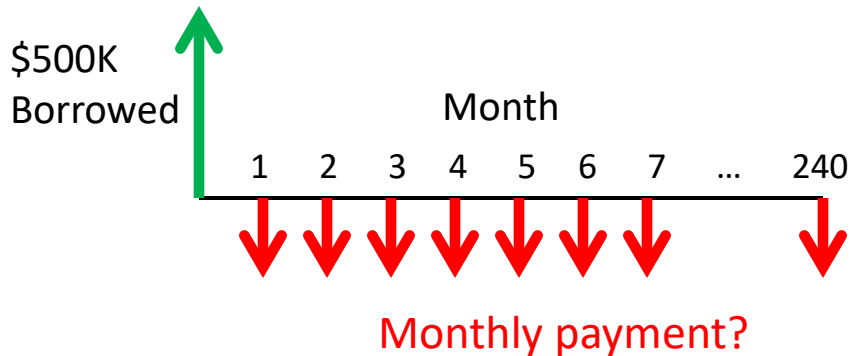
Hint: Interest rates on loans in the US are stated as APRs.

\$500K borrowed for 20 years
at 5% APR with monthly payments.



Financial registers

240	n		N0		CF0 (R0)
.41667	i		N1		CF1 (R1)
500000	PV		N2		CF2 (R2)
?	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Should be \$3,299.78/month.

Procedure:

1. Clear the registers.
2. Enter n and i using the $12x$ and $12\div$ functions.
3. Enter PV.
4. Press PMT.

exercise

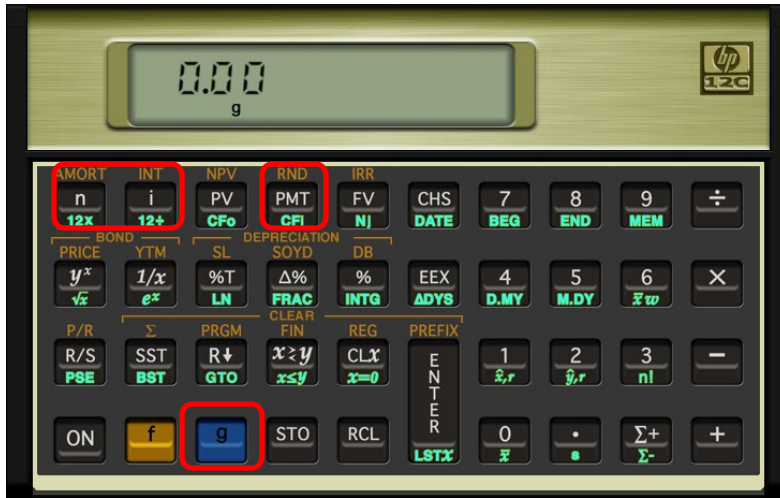
Acme has just made their 120th payment on the mortgage. What is the remaining principal on the loan?

exercise

Acme has just made their 120th payment on the mortgage. What is the remaining principal on the loan?

Hint: Acme has 10 years = 120 payments left and the interest rate is unchanged.

principal remaining after 120th payment



Remaining
principal

Month

1 2 3 4 5 6 7 ... 120

Monthly payment = \$3,299.78

Remaining principal should be \$311,107.59.

Financial registers

120	n		N0		CF0 (R0)
.41667	i		N1		CF1 (R1)
?	PV		N2		CF2 (R2)
-3299.78	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)

Procedure:

1. Clear the registers.
2. Enter n and i using the $12x$ and $12\div$ functions.
3. Enter PMT.
4. Press PV.

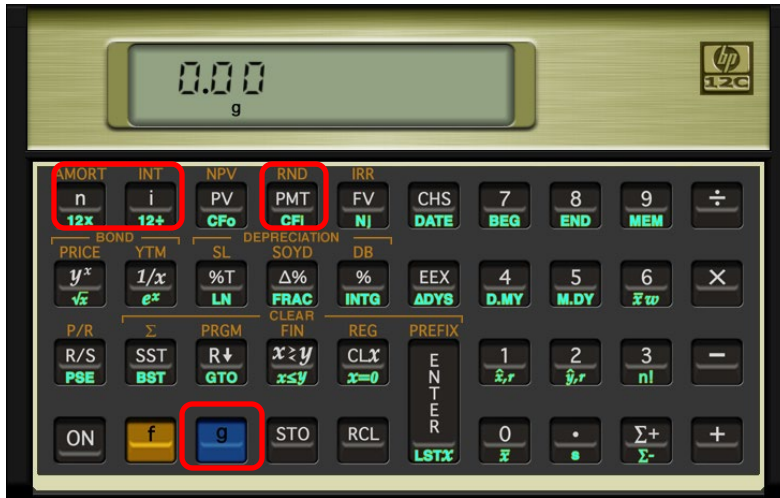
exercise

Ten years ago, Acme Corporation financed a new warehouse in part with a mortgage for \$500K for 20 years at 5%.

A year from now, Acme will have just made their 132nd payment. Between now and then, how much interest will Acme have paid?

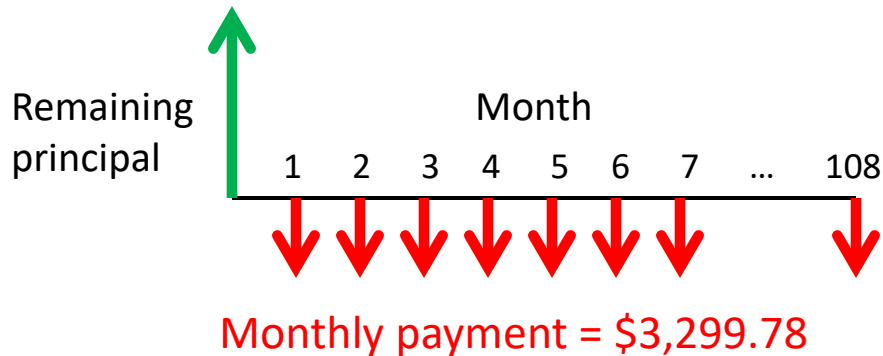
After the 132nd payment, Acme will have $240 - 132 = 108$ payments left.

principal remaining after 132nd payment



Financial registers

108	n		N0		CF0 (R0)
.41667	i		N1		CF1 (R1)
?	PV		N2		CF2 (R2)
-3299.78	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Procedure:

1. Clear the registers.
2. Enter n and i using the 12x and 12÷ functions.
3. Enter PMT.
4. Press PV.

Remaining principal should be \$286,506.94.

after 132nd payment

Remaining principal = \$286,506.94

Amount of principal paid off in the year =

Remaining principal after 120th payment –

Remaining principal after 132nd payment

= 311,107.59 – 286,506.94 = \$24,600.65

Total of payments = 12 * 3299.78 = \$39,597.36

Interest paid = Total payments – Principal paid off

= 39,597.36 – 24,600.65 = \$14,996.69

amortization

amortize

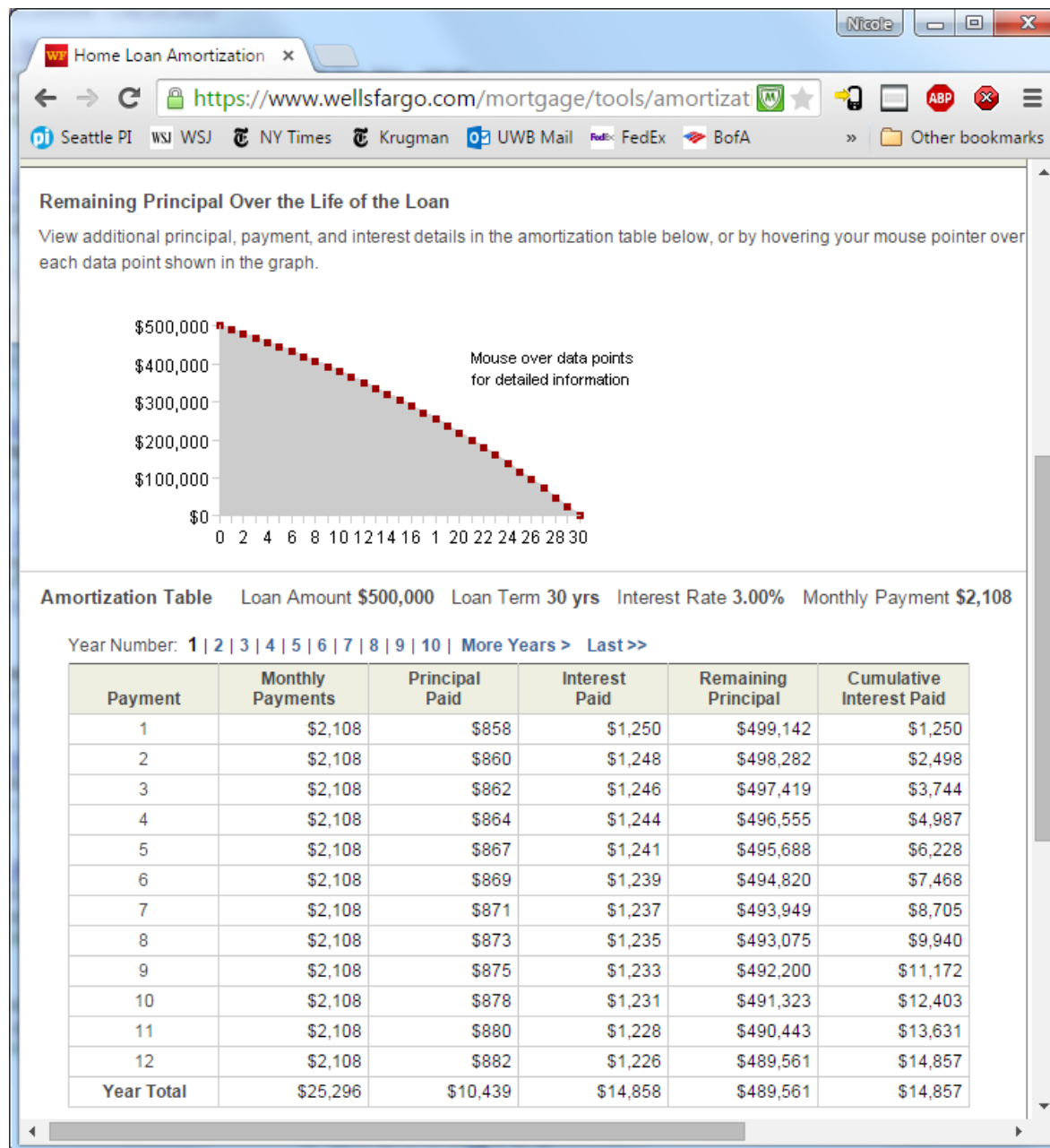
A term used in lending that means each payment will be allocated to separate principal and interest amounts.

The allocation is calculated to the *nearest unit, usually a penny*.

When the remaining principal goes to zero, the loan is *fully amortized*.

Most commonly used in mortgage loans.

Typically presented as a table called an *amortization schedule*.



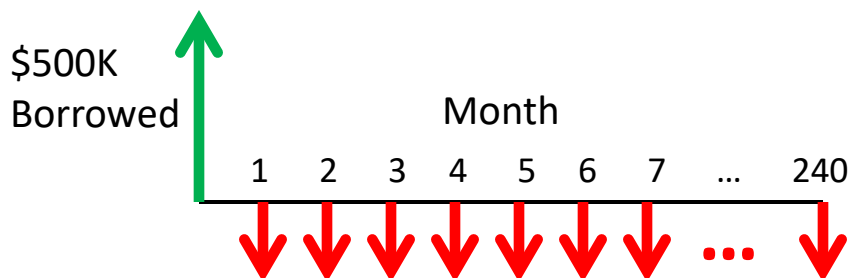
<https://www.wellsfargo.com/mortgage/tools/amortization/>

amortization function



Financial registers

0	n		N0		CF0 (R0)
.41667	i		N1		CF1 (R1)
500000	PV		N2		CF2 (R2)
-3299.78	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Monthly payment = \$3,299.78

How much interest and principal has been paid by the 120th payment?

Procedure:

1. Clear the registers.
2. Enter PV, PMT, n and i.
3. Enter number of payments to amortize.
4. Press AMORT.
5. x contains interest paid
6. y contains principal paid
7. RCL PV to get principal remaining
8. RCL n to get number of payments amortized so far.

principal remaining after 120th payment

Slight difference between:

1. PV of remaining payments = \$311,107.59
2. Amortized remaining principal = \$311,107.41

Difference is because under amortization, amounts allocated to principal versus interest are calculated only to the *nearest penny* (or whatever digits of precision you've set via f-key.)

(A bank will care about this difference, I don't.)

creating an amortization schedule

Because:

1. Principal paid is subtracted from PV.
2. Number of payments amortized added to n.

The amortization function can be used to be used to amortize successive payments.

1. PV always contains remaining principal.
2. n always contains number of payment amortized so far.
3. Works with loans that never fully amortize, e.g., because of a balloon payment at the end.

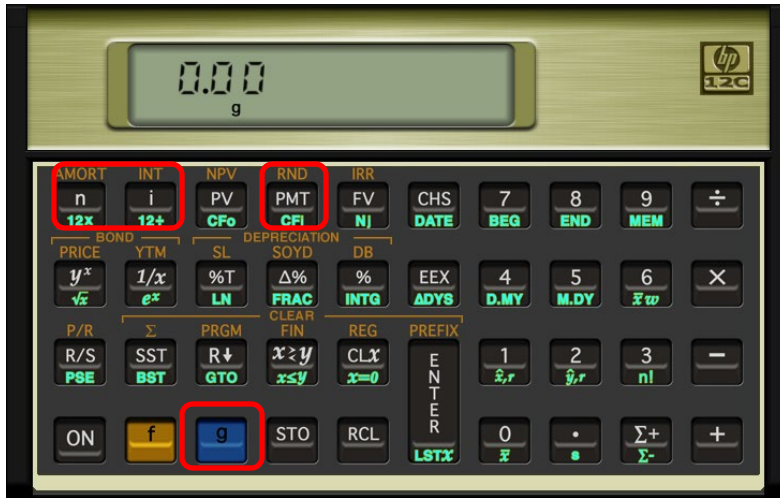
exercise

You've just bought a home with \$400K, 30-year mortgage at 3% APR.

1. What does the cash flow diagram look like?
2. What is your monthly payment?
3. Fill in this table for the first 3 years:

Year	Beginning principal	Principal paid	Interest paid	Ending principal
1	400,000			
2				
3				
Total paid				

\$400K borrowed for 30 years
at 3% APR with monthly payments.



\$400K
Borrowed

Month

1 2 3 4 5 6 7 ... 360

Monthly payment?

Should be \$1,686.42/month.

Financial registers

360	n		N0		CF0 (R0)
.25	i		N1		CF1 (R1)
400000	PV		N2		CF2 (R2)
?	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)

Procedure:

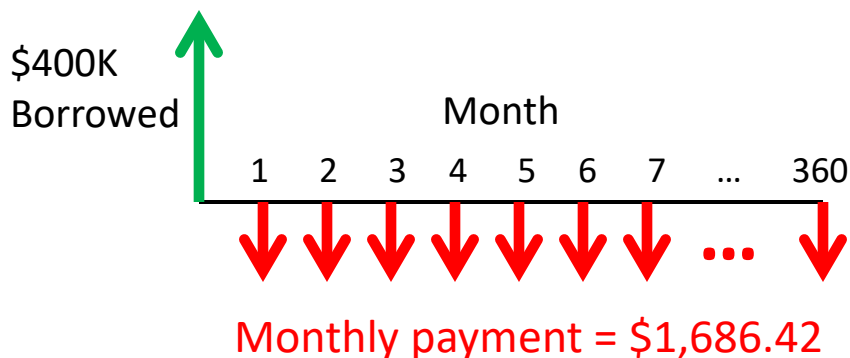
1. Clear the registers.
2. Enter n and i using the 12x and 12÷ functions.
3. Enter PV.
4. Press PMT.

amortize first year



Financial registers

0	n		N0		CF0 (R0)
.25	i		N1		CF1 (R1)
400000	PV		N2		CF2 (R2)
-1686.42	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Interest paid = \$11,885.80
Principal paid = \$8,351.24
Principal remaining = \$391,648.76

Procedure:

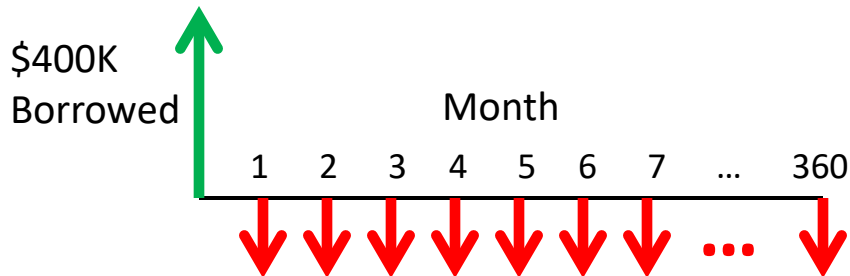
1. Set $n = 0$.
2. Enter 12 payments to amortize.
3. Press AMORT.
4. x contains interest paid
5. y contains principal paid
6. RCL PV to get principal remaining

amortize second year



Financial registers

12	n		N0		CF0 (R0)
.25	i		N1		CF1 (R1)
391648.76	PV		N2		CF2 (R2)
-1686.42	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Monthly payment = \$1,686.42

Interest paid = \$11,631.78

Principal paid = \$8,605.26

Principal remaining = \$383,043.50

Procedure:

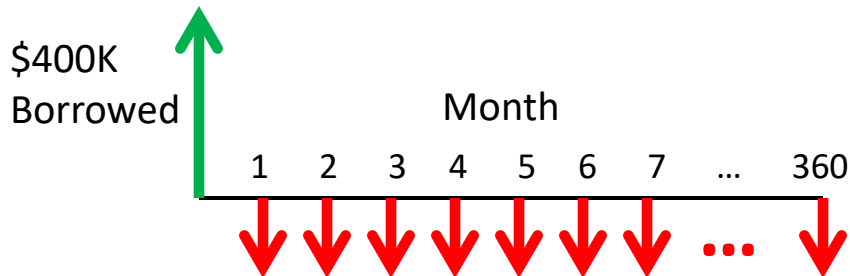
1. Enter 12 payments to amortize.
2. Press AMORT.
3. x contains interest paid
4. y contains principal paid
5. RCL PV to get principal remaining

amortize third year



Financial registers

24	n		N0		CF0 (R0)
.41667	i		N1		CF1 (R1)
383043.50	PV		N2		CF2 (R2)
-1686.42	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Monthly payment = \$1,686.42

Interest paid = \$11,370.04

Principal paid = \$8,867.00

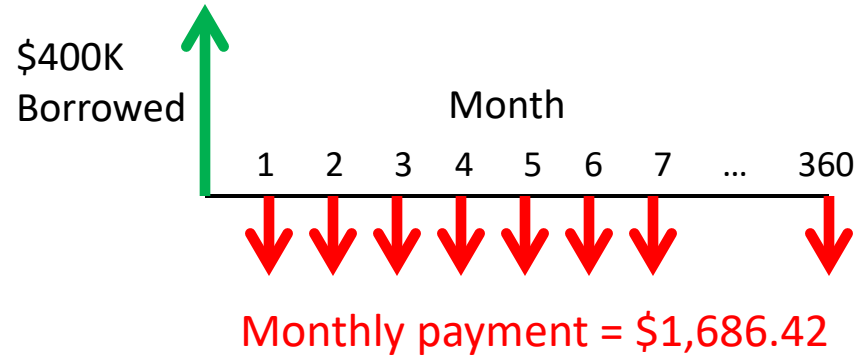
Principal remaining = \$374,176.50

Procedure:

1. Enter 12 payments to amortize.
2. Press AMORT.
3. x contains interest paid
4. y contains principal paid
5. RCL PV to get principal remaining

exercise

You've just bought a home with \$400K, 30-year mortgage at 3% APR.



1. What does the cash flow diagram look like?
2. What is your monthly payment?
\$1,686.42
3. Fill in this table for the first 3 years:

Year	Beginning principal	Principal paid	Interest paid	Ending principal
1	400,000	8,351.24	11,885.80	391,648.76
2	391,648.76	8,605.26	11,631.78	383,043.50
3	383,043.50	8,867.00	11,370.04	374,176.50
Total paid		25,823.50	34,887.62	

escrow

Usually when you get a mortgage, your payment will be more than principal + interest.

Usually, the bank will want you to *escrow* fire insurance and property tax payments.

You pay an amount each month estimated to cover insurance and tax.

The mortgage company then makes those payments.

This way, the mortgage company knows for sure that the payments got made.

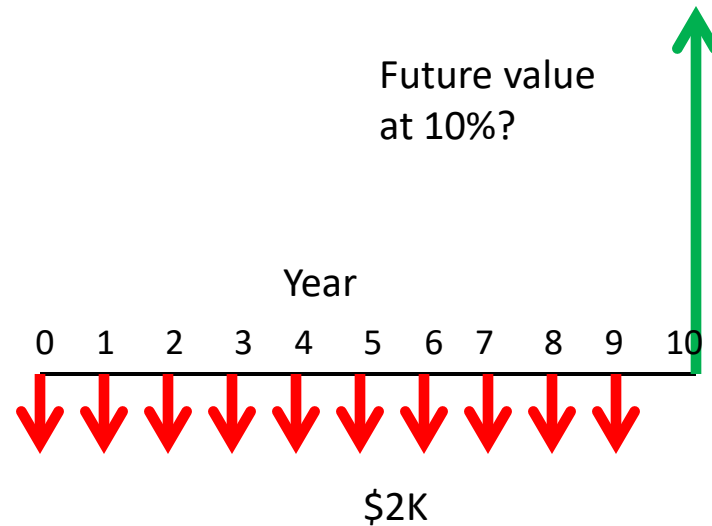
investments

Interest rates on investments are different than interest rates on loans.

Loans are stated as APRs and are compounded monthly.

Investors normally use annual compounding when evaluating an investment.

sinking funds

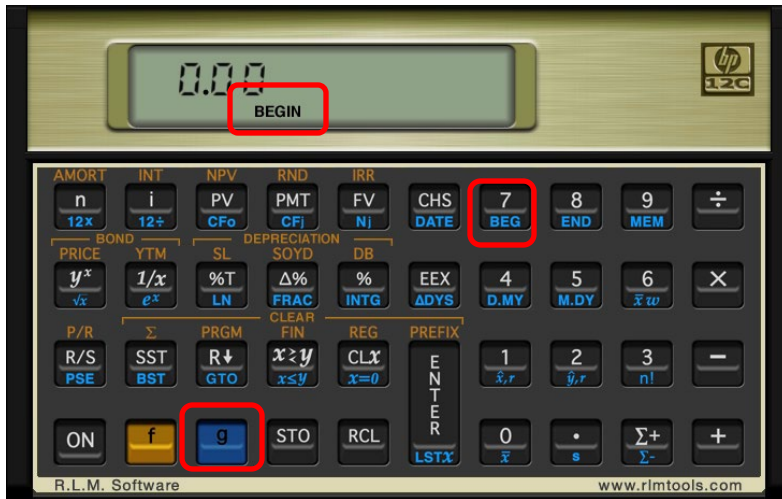


Annuities that you pay into to get a future return are call sinking funds.

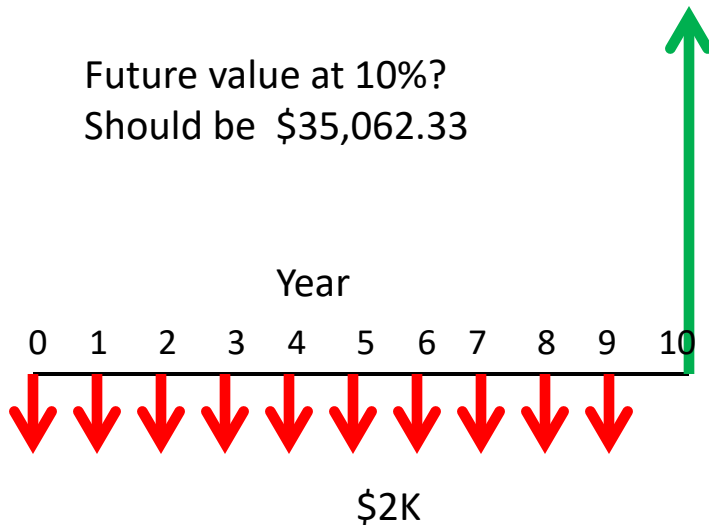
An example would be a regular savings plan.

Annuity payments are at the ***beginning*** of the period, not the end.

sinking fund at 10%



Future value at 10%?
Should be \$35,062.33



Financial registers

10	n		N0		CF0 (R0)
10	i		N1		CF1 (R1)
0	PV		N2		CF2 (R2)
-2K	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)

Procedure:

1. Clear the registers.
2. Set in BEGIN mode.
3. Enter n and i .
4. Press FV.

Net present value (NPV)

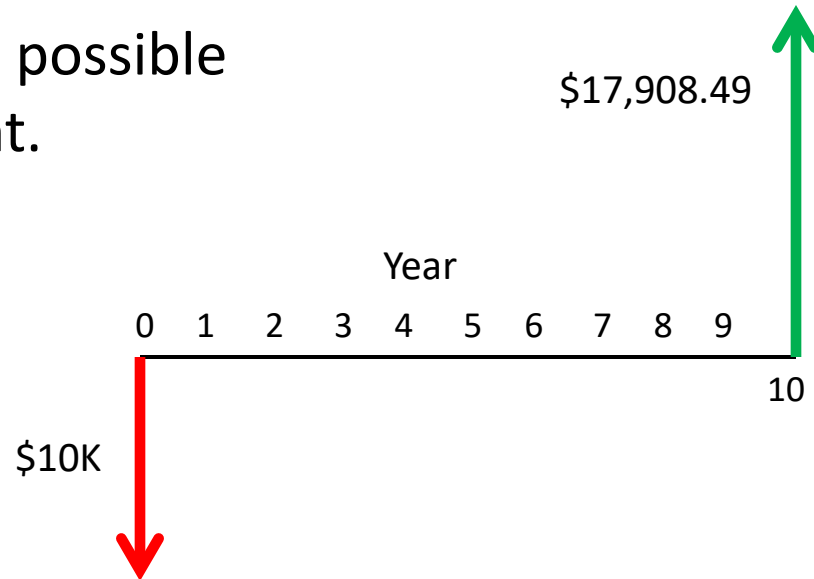
Net sum of all the discounted inflows and outflows.

$$NPV = \sum_{j=0}^n (CF_j) \left(\frac{P}{F} \Big|_{i,j} \right)$$

Could be either positive or negative.

At the ***internal rate of return (IRR)***, the NPV is zero.

Consider this possible investment.



At 6%
$$\frac{P}{F} = \frac{1}{(1+i)^n} = \frac{1}{(1+.06)^{10}} = 0.558395$$

$$NPV = (17,908.49)(0.558395) - 10,000 = 0$$

Similarly,

At 5%
$$NPV = \$994.26$$

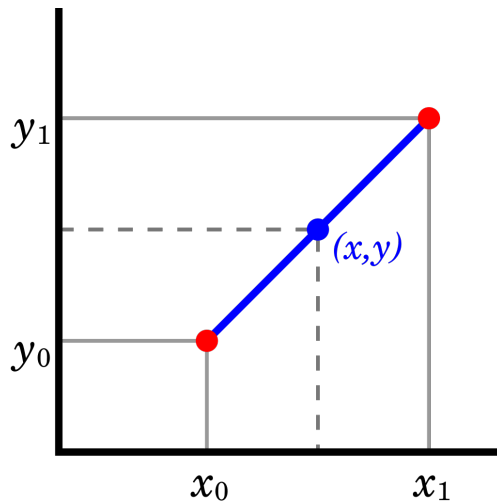
At 7%
$$NPV = -\$896.23$$

calculating IRR

1. Calculate NPV at interest rates expected to be above and below the actual IRR, then interpolate.
2. Use a calculator or a spreadsheet that provides an IRR function.

Simple interpolation

Approximating with a straight line



Choose the points on either side of the point you'd like to estimate.

$$\frac{y - y_0}{x - x_0} = \frac{y_1 - y_0}{x_1 - x_0}$$

Given the desired x:

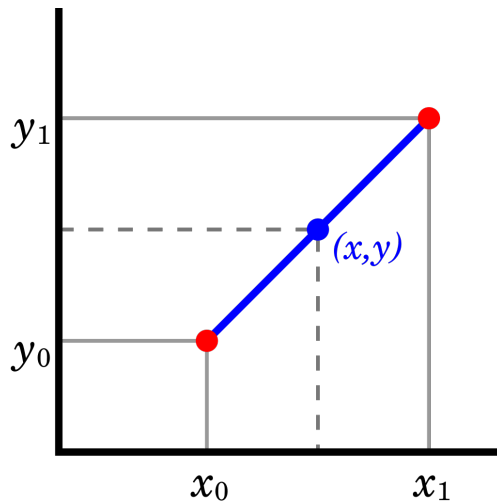
$$y = y_0 + (y_1 - y_0) \left(\frac{x - x_0}{x_1 - x_0} \right)$$

Given the desired y:

$$x = x_0 + (x_1 - x_0) \left(\frac{y - y_0}{y_1 - y_0} \right)$$

Image source: <https://en.wikipedia.org/wiki/File:LinearInterpolation.svg>

interpolating



x = interest rate

y = NPV

$x_0 = 5\%$

$y_0 = 994.26$

$y_1 = -896.23$

$x_1 = 7\%$

Find the x (interest rate) at which y (NPV) = 0

$$x = x_0 + (x_1 - x_0) \left(\frac{y - y_0}{y_1 - y_0} \right)$$

$$x = 5 + (7 - 5) \left(\frac{0 - 994.26}{-896.23 - 994.26} \right) = 6.051854$$

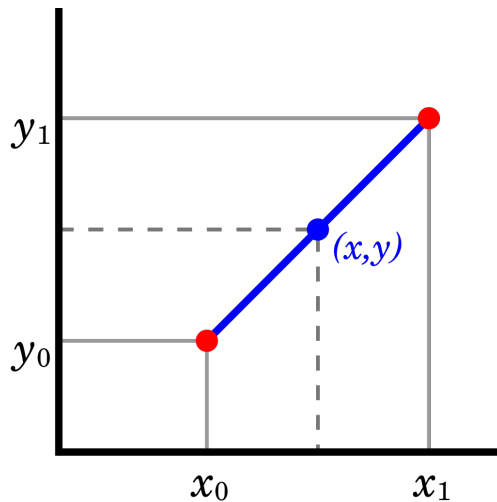
$$\frac{P}{F} = \frac{1}{(1+i)^n} = \frac{1}{(1+.06051854)^{10}} = 0.555670$$

$$\begin{aligned} NPV &= (17,908.49)(0.555670) - 10,000 \\ &= -\$48.780528 \end{aligned}$$

At 5% NPV = \$994.26

At 7% NPV = -\$896.23

next iteration



x = interest rate

y = NPV

$x_0 = 5\%$

$y_0 = 994.26$

$y_1 = -48.780528$

$x_1 = 6.0518954\%$

Find the x (interest rate) at which y (NPV) = 0

$$x = x_0 + (x_1 - x_0) \left(\frac{y - y_0}{y_1 - y_0} \right)$$

$$x = 5 + (6.051854 - 5) \left(\frac{0 - 994.26}{-48.780528 - 994.26} \right) = 6.002661$$

At 5% NPV = \$994.26

At 6.051854%

NPV = -\$48.780528

$$\frac{P}{F} = \frac{1}{(1+i)^n} = \frac{1}{(1+.06002661)^{10}} = 0.558255$$

$$\begin{aligned} NPV &= (17,908.49)(0.558255) - 10,000 \\ &= -\$2.503025 \end{aligned}$$

hurdle rates

For a consumer, the cost of funds is whatever interest rate they're paying on debt.

But big corporations will often establish ***hurdle rates***.

A hurdle rate is the minimum expected IRR the corporation expects from projects it will fund.

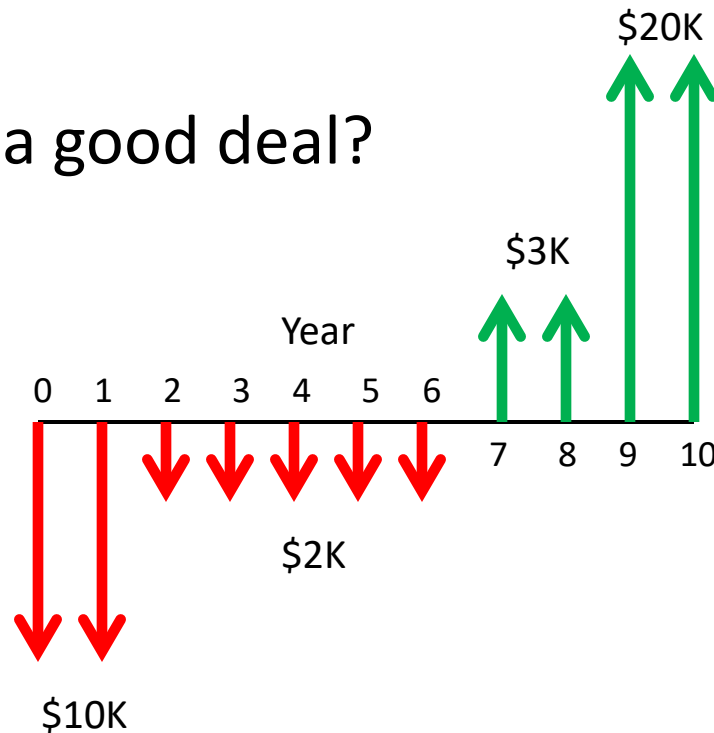
Hurdle rates are intended to be related to the firm's cost of capital, overall risk and other factors.

A typical corporate hurdle rate is 12% but could be 30% or higher.

irregular cash flows

When the cash flows in/out aren't uniform like an annuity, it's called an irregular cash flow.

Is this a good deal?

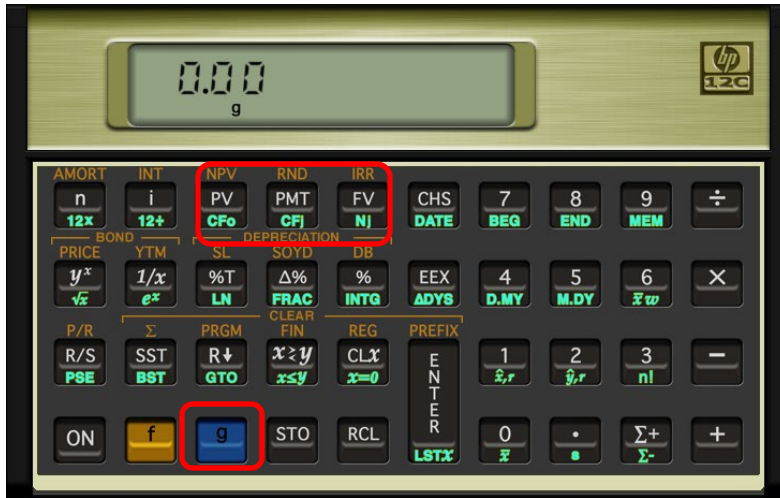


Is this a good deal?

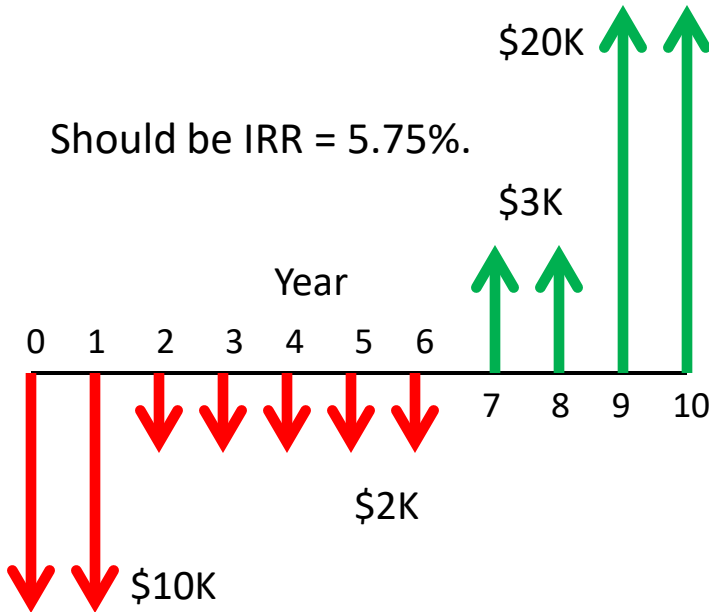


j	Cf_j	N_j
0	-10,000.00	1
1	-10,000.00	1
2	-2,000.00	5
3	3,000.00	2
4	20,000.00	2

IRR for an irregular cash flow.



Should be IRR = 5.75%.



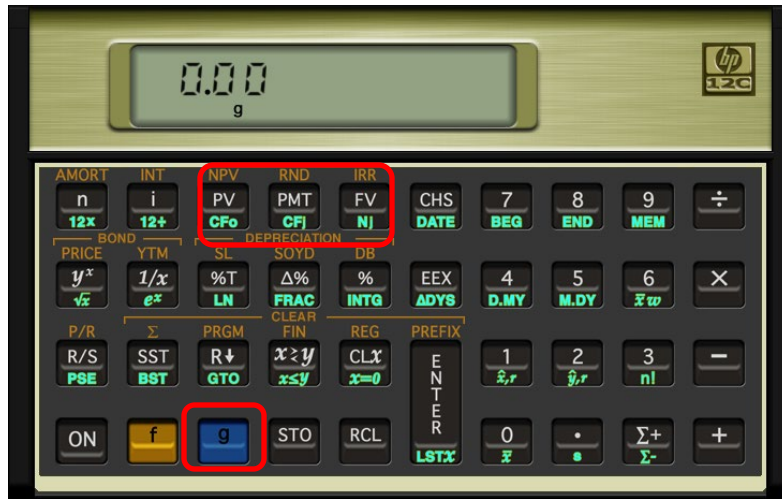
Financial registers

0	n	1	N0	-10K	CF0 (R0)
0	i	1	N1	-10K	CF1 (R1)
0	PV	5	N2	-2K	CF2 (R2)
0	PMT	2	N3	3K	CF3 (R3)
0	FV	2	N4	20K	CF4 (R4)
	:		:		
			N20		CF20 (R.9)

Procedure:

1. Clear the registers.
2. CF0 = -10,000.
3. Enter CF1 = -10,000, N1 = 1.
4. Repeat for CF2 thru CF4.
5. Press IRR.

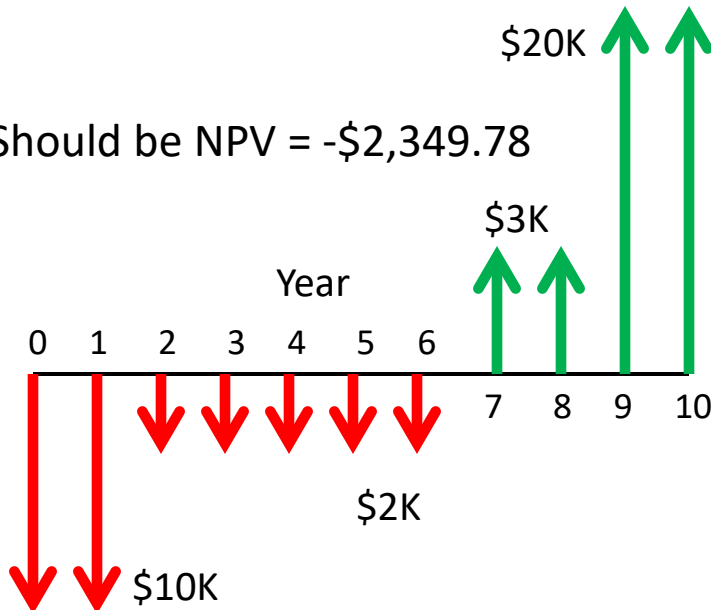
NPV for an irregular cash flow at 7%



Financial registers

0	n	1	N0	-10K	CF0 (R0)
0	i	1	N1	-10K	CF1 (R1)
0	PV	5	N2	-2K	CF2 (R2)
0	PMT	2	N3	3K	CF3 (R3)
0	FV	2	N4	20K	CF4 (R4)
		:	:		
			N20		CF20 (R.9)

Should be NPV = -\$2,349.78



Procedure:

1. Clear the registers.
2. CF0 = -10,000.
3. Enter CF1 = -10,000, N1 = 1.
4. Repeat for CF2 thru CF4.
5. Enter i = 7%.
6. Press NPV

comparing alternatives

1. Calculate the NPV of each alternative at your discount rate, then choose the one with highest NPV.
2. Calculate the IRR for each alternative, then choose the one with the highest IRR.
3. Calculate the NPV of the differences between the two alternatives.

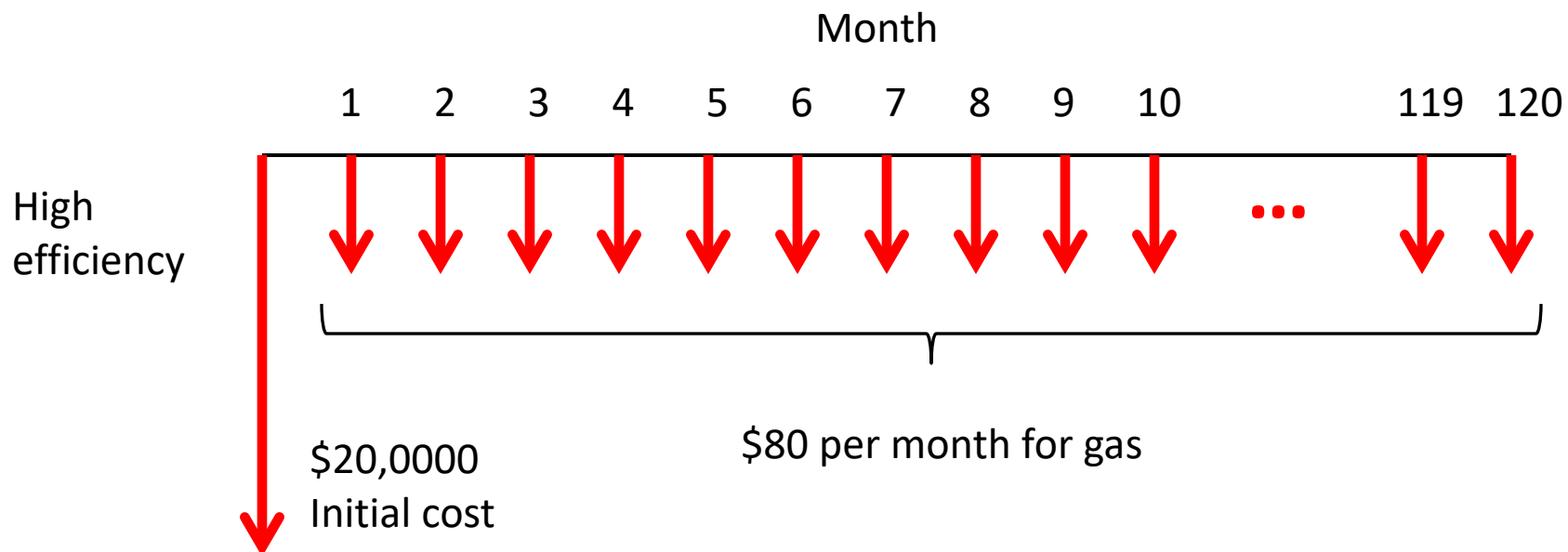
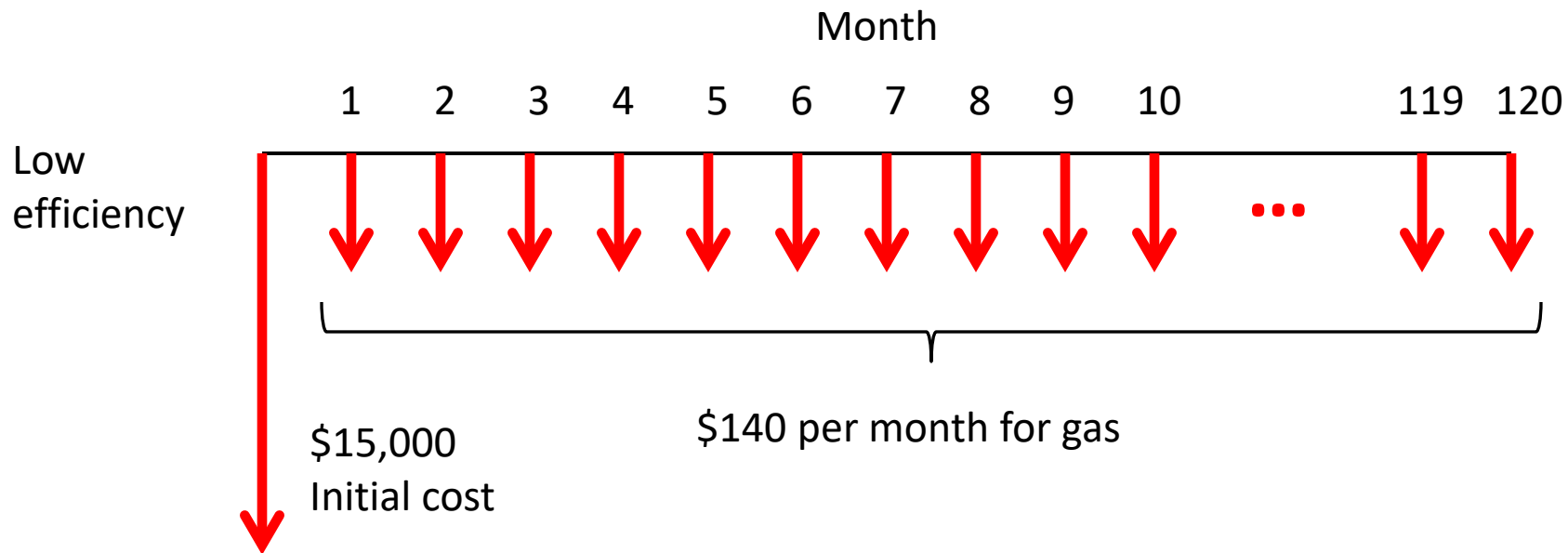
exercise

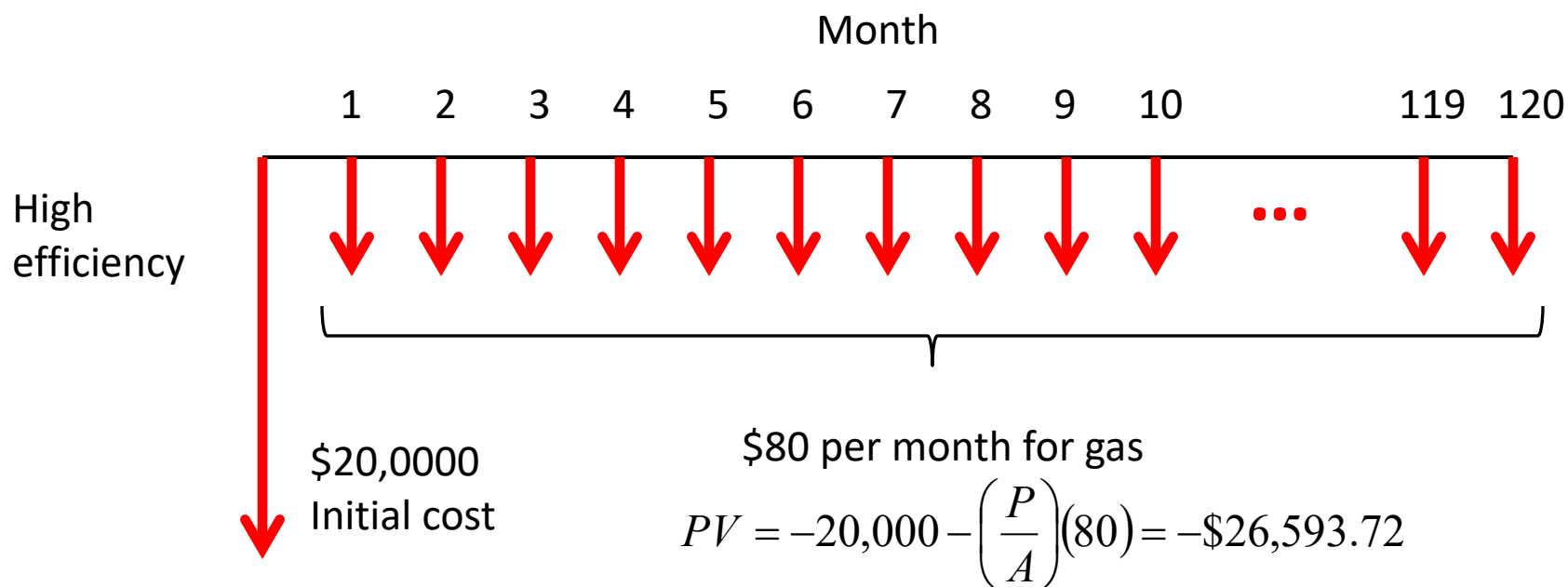
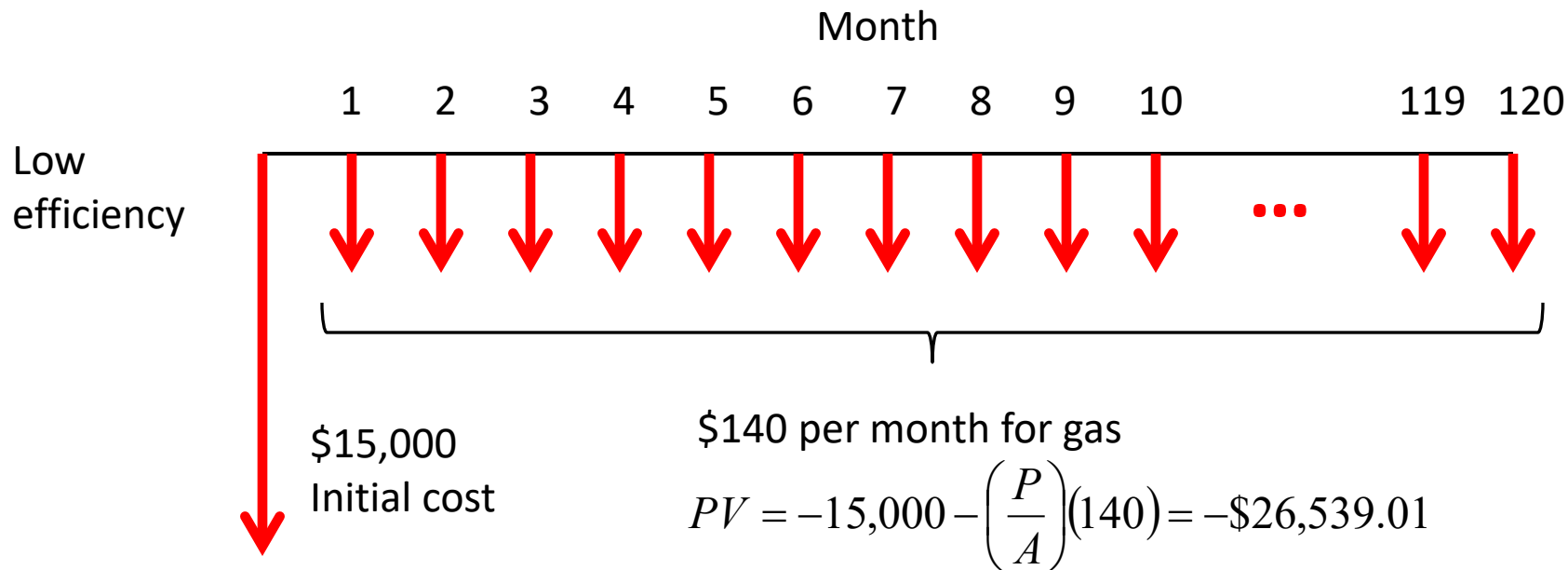
You need to replace your furnace. You have two options.

1. For \$15K, you can buy a low efficiency unit that will cost you \$140/mo in gas.
2. For \$20K, you can buy a high efficiency unit that will cost you \$80/mo in gas.

You expect either will have a 10 year lifetime.

1. Draw the two cash flow diagrams.
2. You expect to pay for the new furnace with a loan at 8% APR. Which would you choose?





In this completely made-up example, the low efficiency furnace is slightly cheaper:

$$\$26,593.72 - 26,539.07 = \$54.72 \text{ savings}$$

If the difference was that small, a lot of people would probably choose high efficiency.

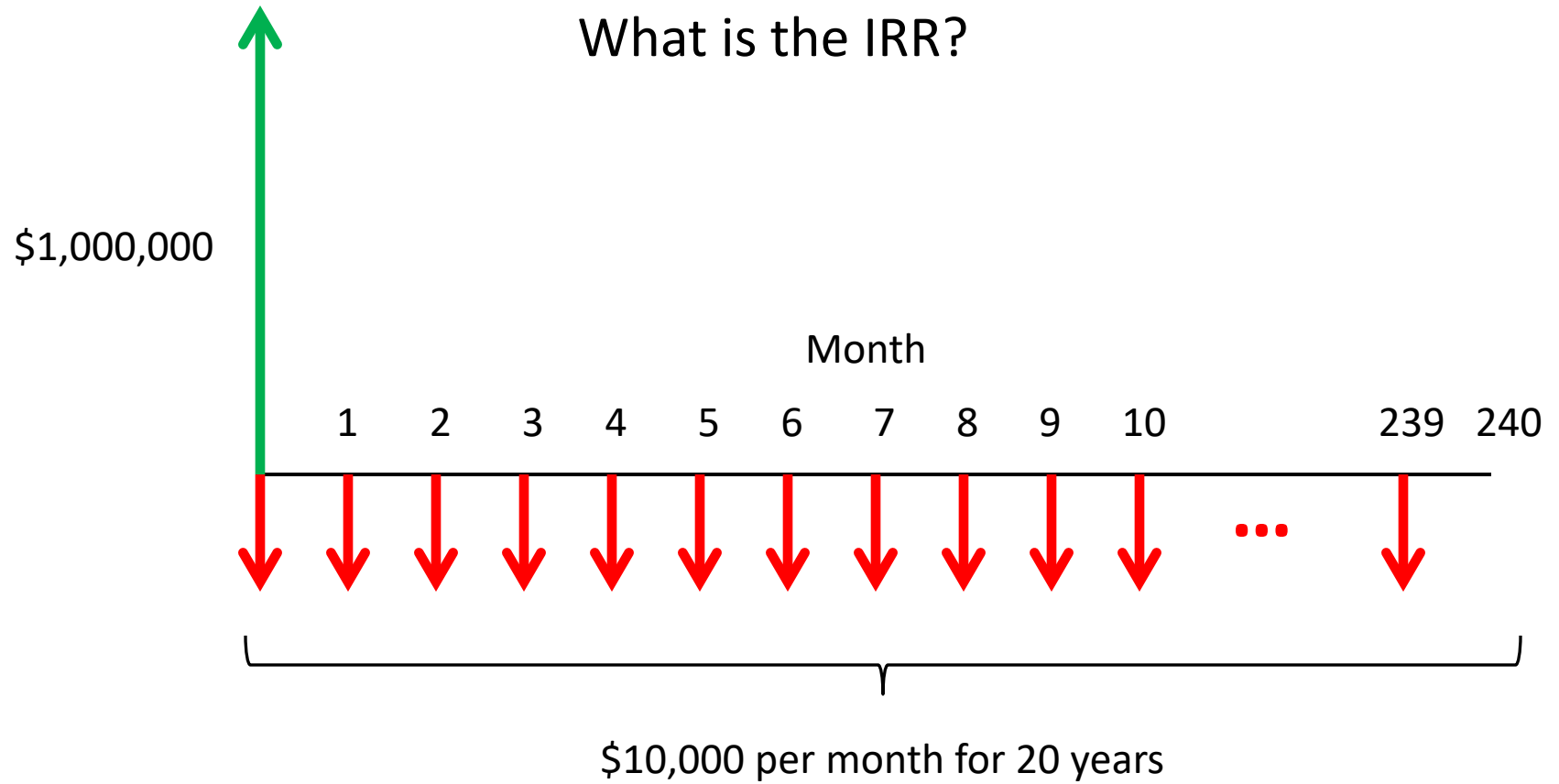
1. Risk of energy costs rising,
2. Resale value
3. Social consciousness.

exercise

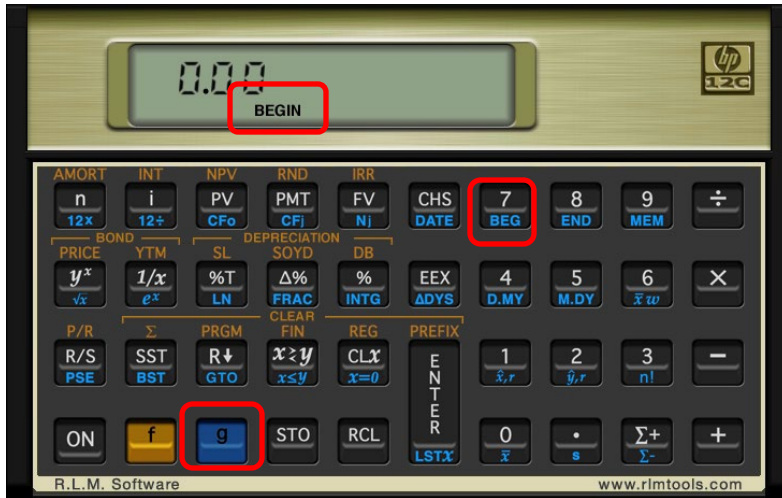
Ed McMahon has risen from the dead and is standing on your doorstep, offering you a check for \$1,000,000 now or \$10,000/month for 20 years. Tax-free.

Which is the better deal?

What is the IRR?

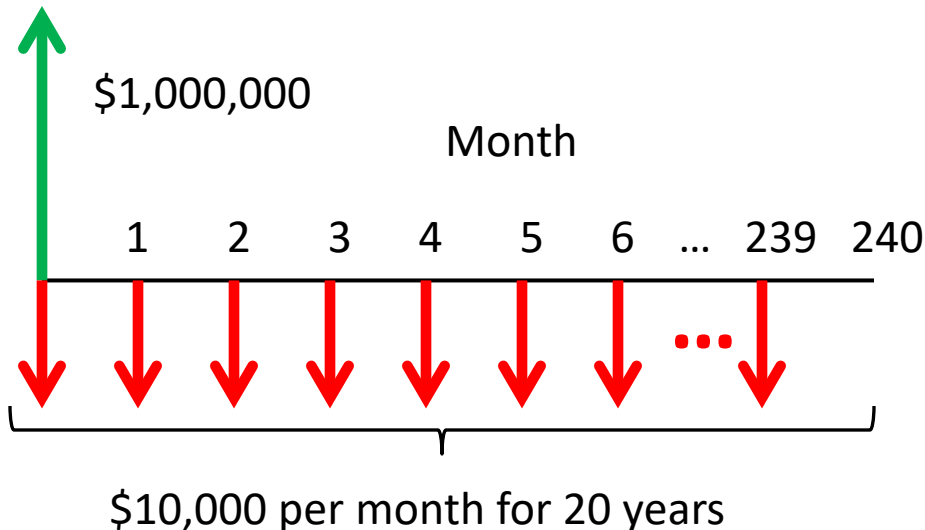


finding IRR



Financial registers

240	n		N0		CF0 (R0)
	i		N1		CF1 (R1)
1M	PV		N2		CF2 (R2)
-10K	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)



Procedure:

1. Clear the registers.
2. Set in BEGIN mode.
3. Enter n = 240.
4. Enter PV and PMT.
5. Press i (rate per month)

depends on your discount rate

IRR = 10.66% APR = 11.19% effective

At 8%, the annuity is worth \$1,203,513.20

At 12%, the annuity is worth \$917,276.11

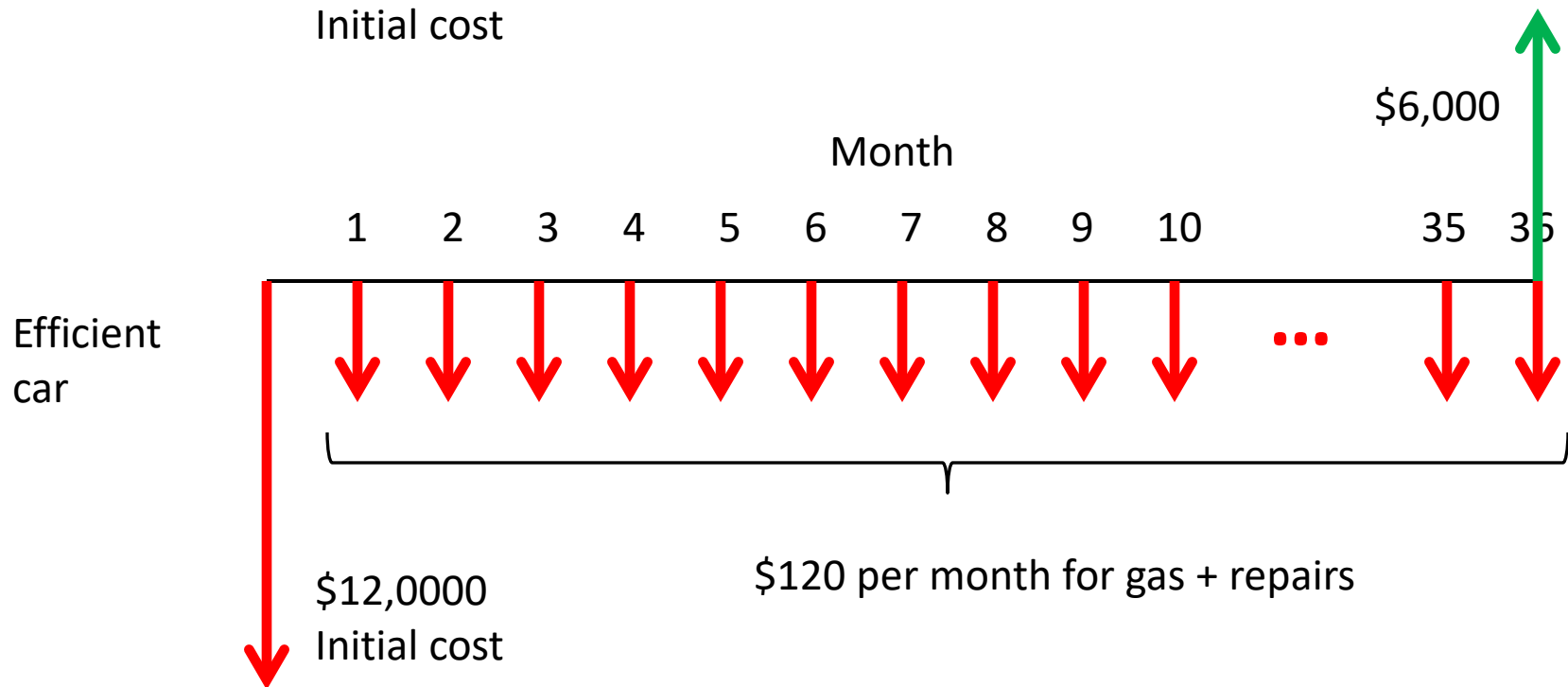
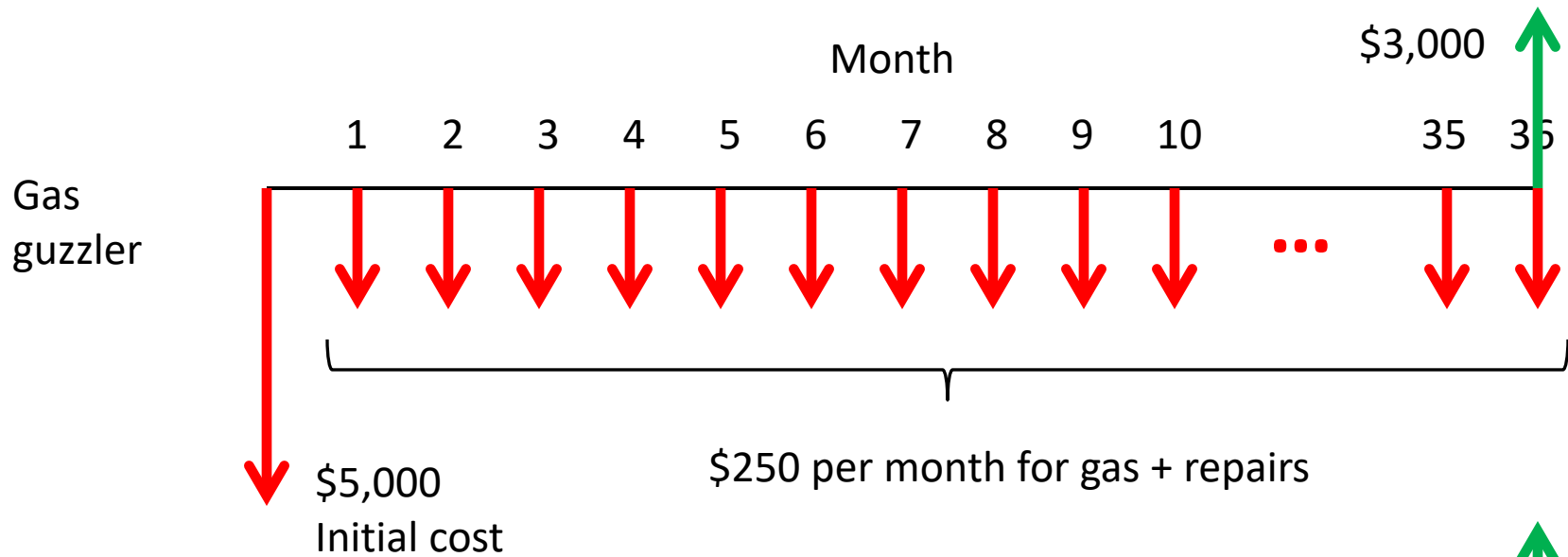
If you prefer cash now, you'll choose the \$1M.

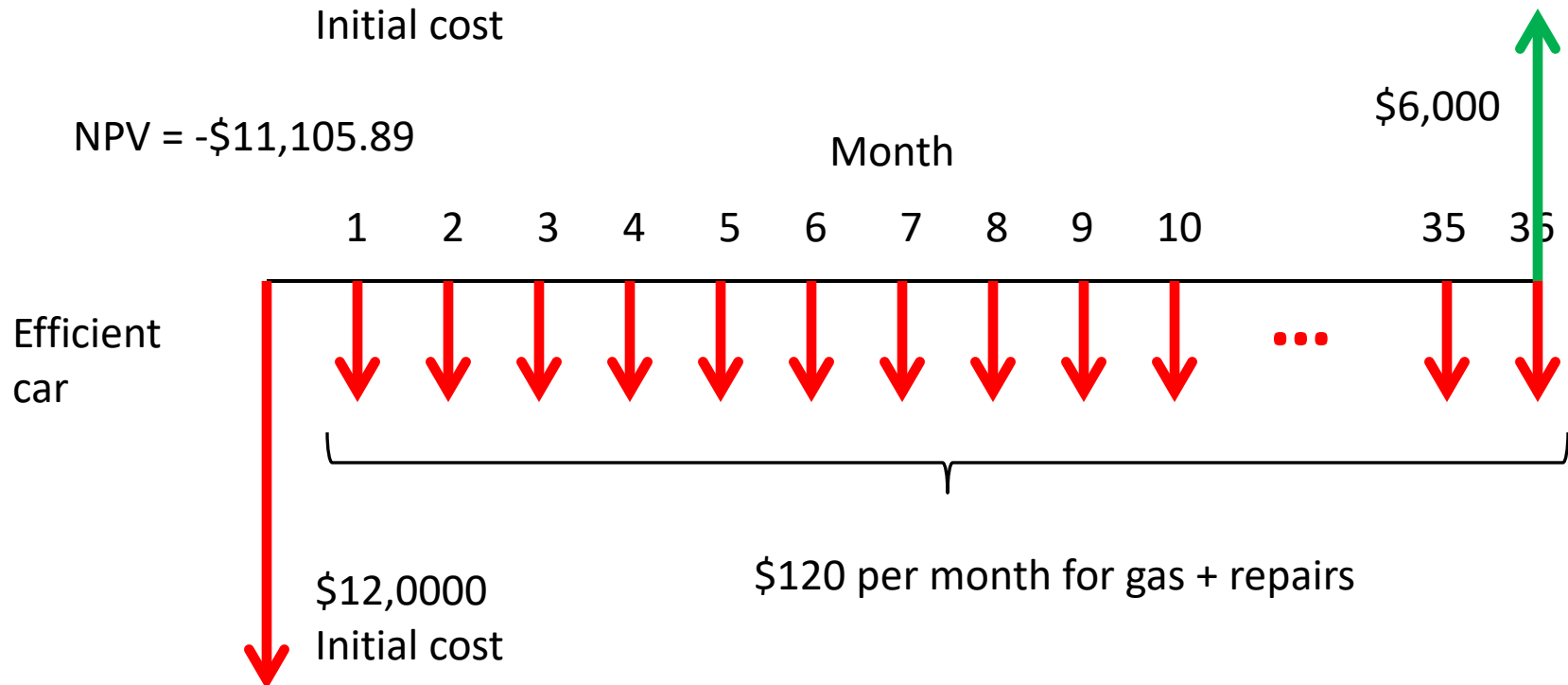
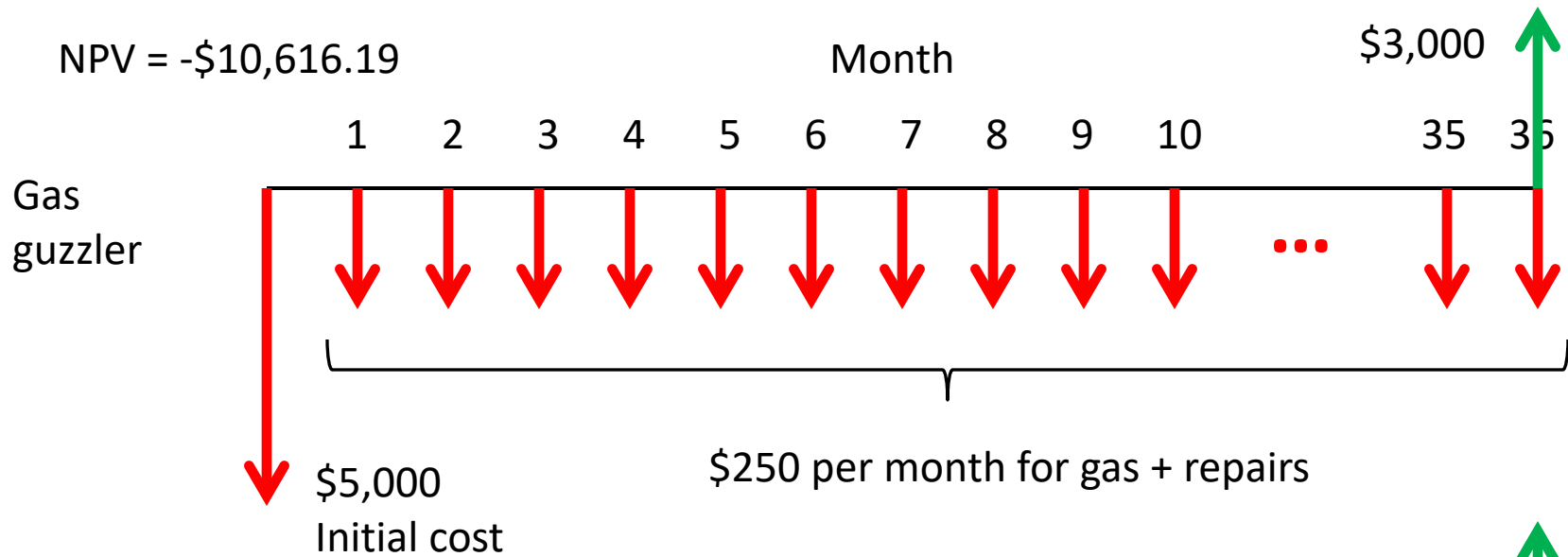
exercise

You need a car for the next 3 years and you're considering two alternatives.

1. An older gas guzzler for \$5,000 which you estimate will require \$150/month for gas and \$100/month on repairs. After 3 years, you expect it could be sold for \$3,000.
2. A newer, more efficient car that for \$12,000 that would need only \$80/month in gas and \$40/ month for repairs. After 3 years, you expect it could be sold for \$6,000.

You're living on money you're borrowing at 8% APR. Which is the better deal?





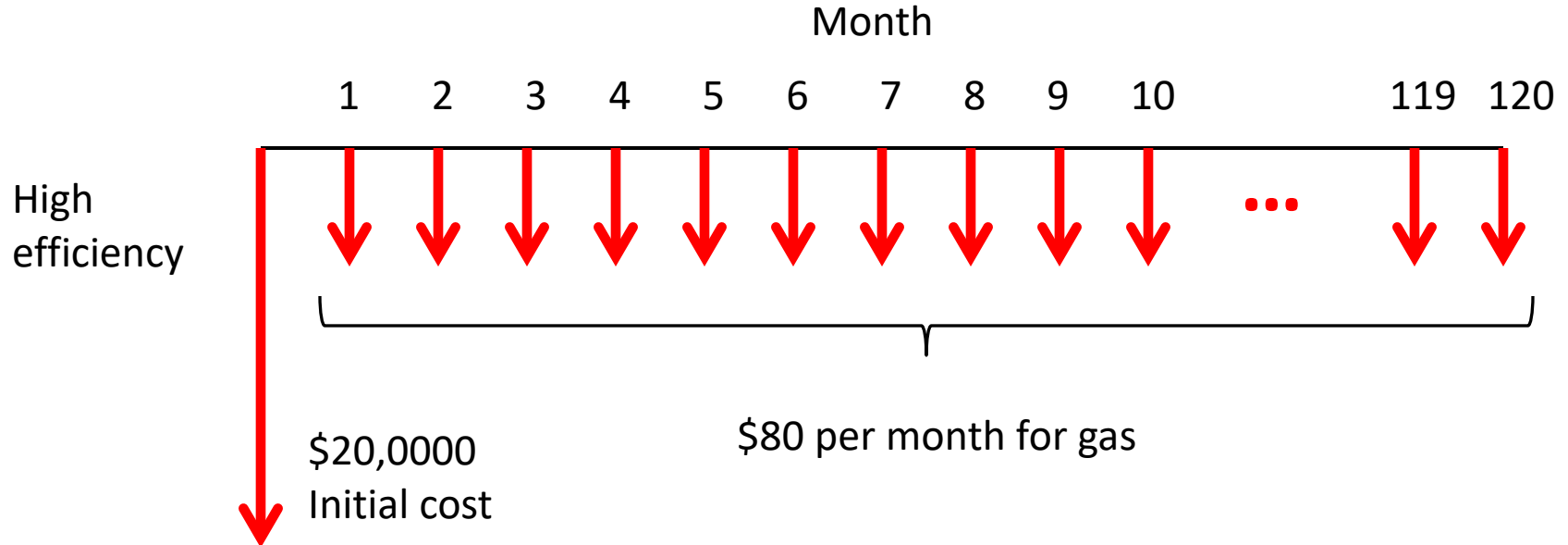
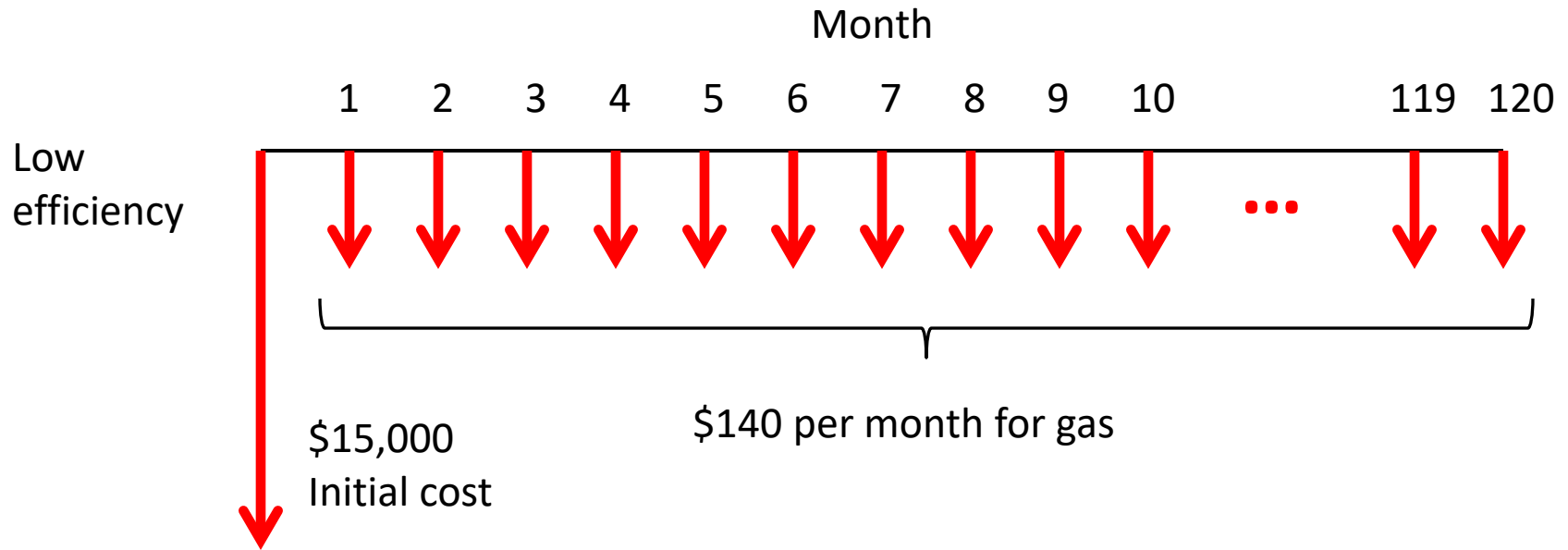
exercise

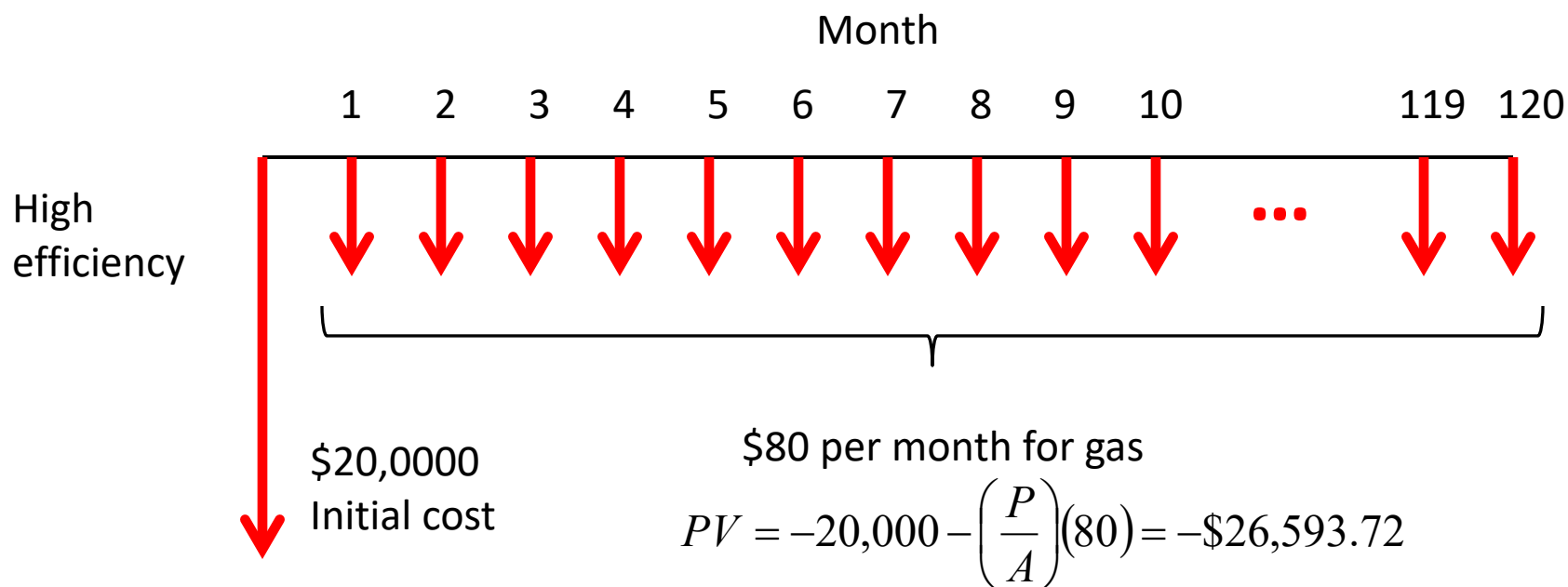
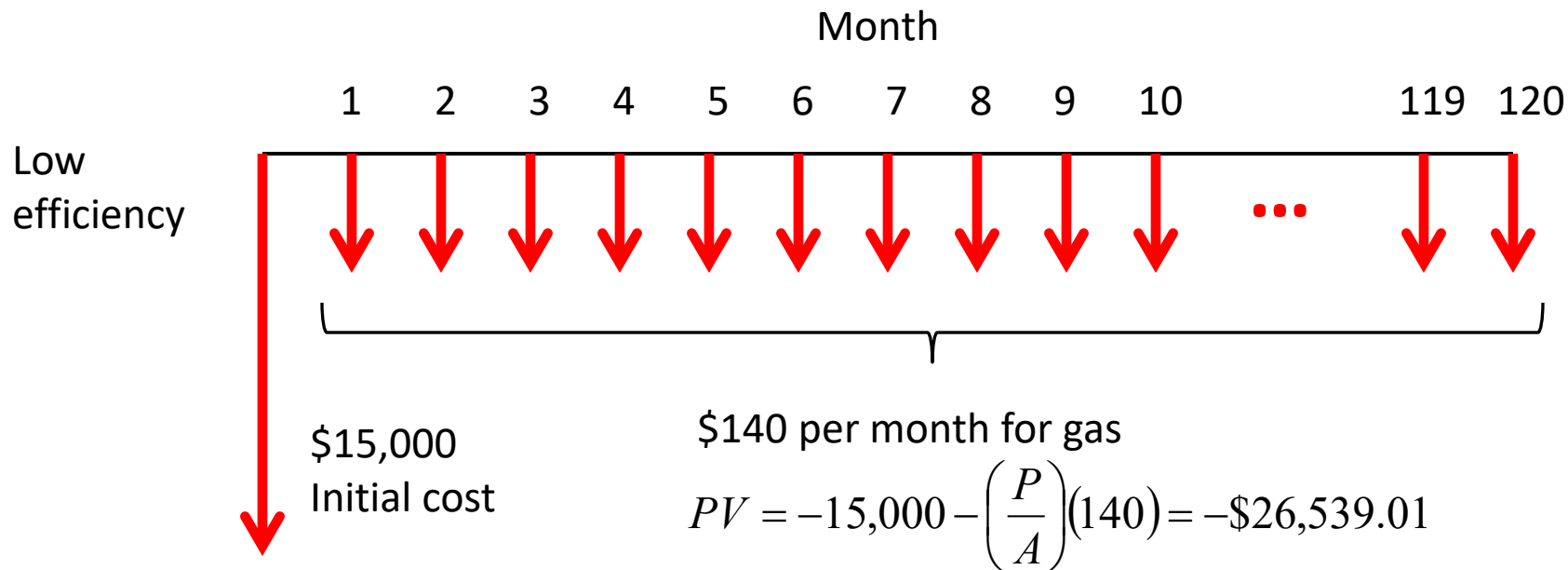
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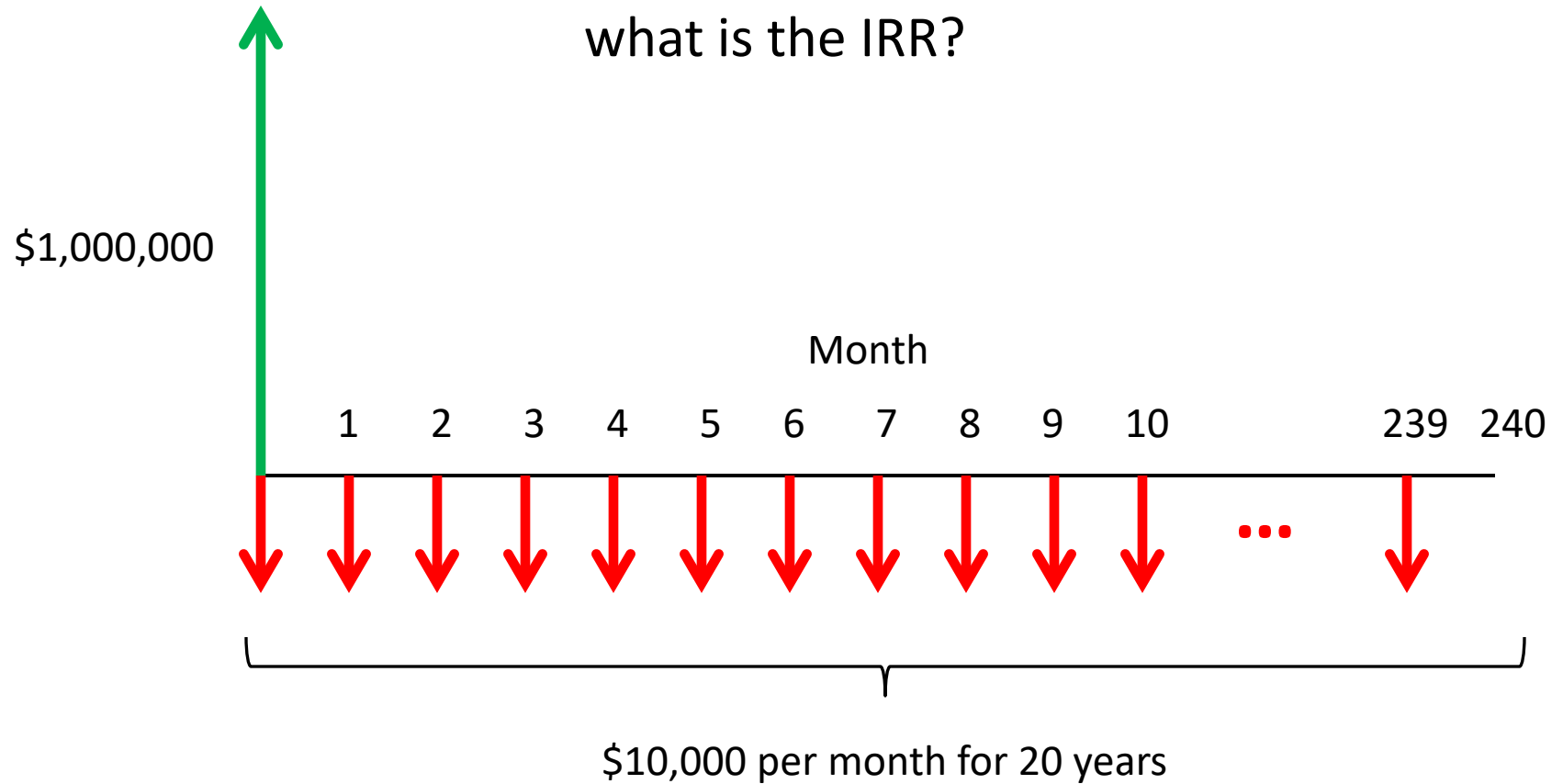
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2. Resale value
3. Social consciousness.

exercise

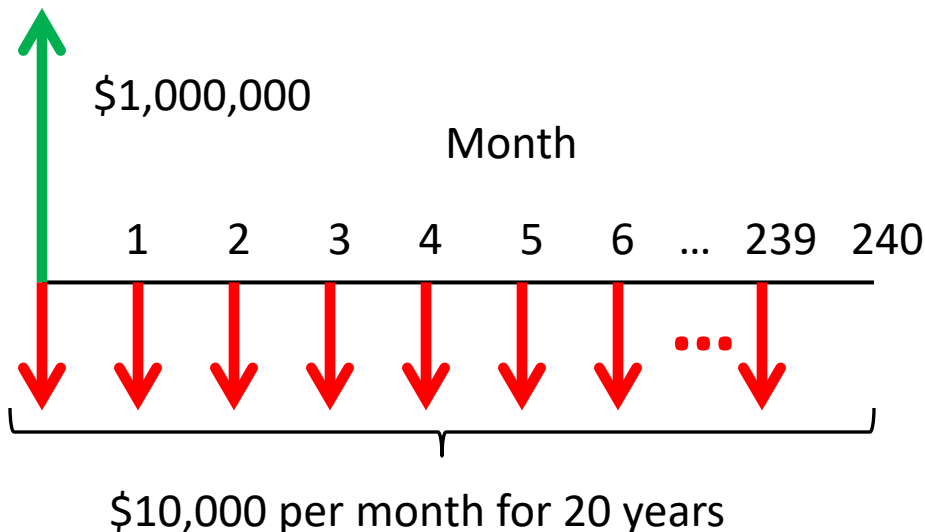
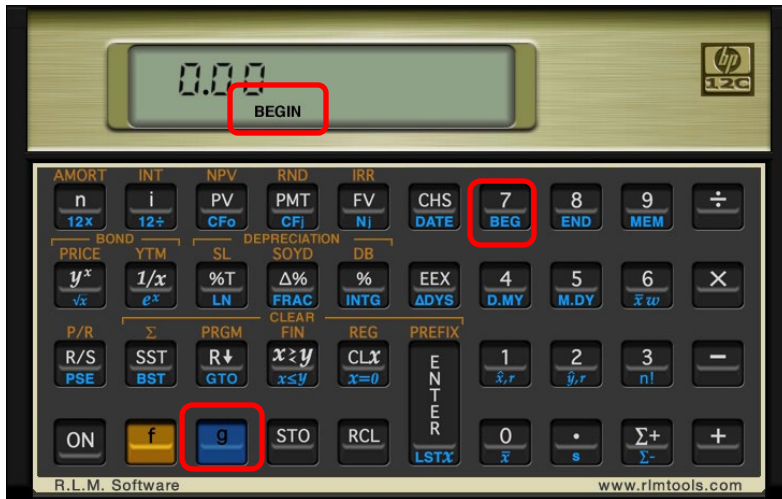
Ed McMahon has risen from the dead and is standing on your doorstep, offering you a check for \$1,000,000 now or \$10,000/month for 20 years. Tax-free.

Which is the better deal?

what is the IRR?



finding IRR



Financial registers

240	n		N0		CF0 (R0)
	i		N1		CF1 (R1)
1M	PV		N2		CF2 (R2)
-10K	PMT		N3		CF3 (R3)
0	FV		N4		CF4 (R4)
		:		:	
			N20		CF20 (R.9)

Procedure:

1. Clear the registers.
2. Set in BEGIN mode.
3. Enter $n = 240$.
4. Enter PV and PMT.
5. Press i (rate per month)